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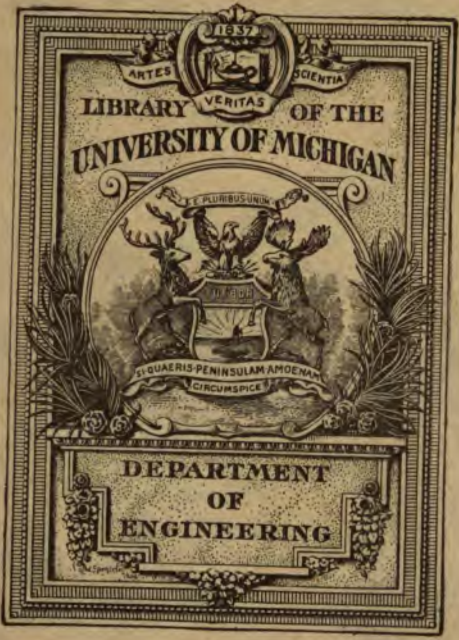

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COMPILED AND ARRANGED AS A TEXT BOOK

FOR THE USE OF CADETS AT THE U. S.
NAVAL ACADEMY,

BY

odney
R. R. INGERSOLL,
Lieut.-Commander, U. S. N.

THIRD EDITION.
REVISED AND PARTIALLY REWRITTEN.

1893.

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PREFACE.

This volume aims to present, in a simple form, for the purpose of the instruction of Naval Cadets, methods for the solution of the various problems in Ballistics which are likely to be met with in Naval Gunnery Practice.

For a more elaborate discussion of the motion of projectiles in a resisting medium, as well as for different methods for the computing of trajectories, the reader is referred to such works as those of Mayevski, Bashforth, Siacci, and the recent complete and valuable work of Captain James Ingalls, U. S. Artillery, whose works have been consulted in the preparation of the following pages.

Only one method for the complete solution of trajectories in air is here presented, on account of the limited time available for instruction at this institution in this branch, and also for the reason that the method selected will be found sufficiently accurate in practice for work with naval guns, while the labor of computation involved is very much less than is necessary for the same accuracy with many other methods.

The method adopted is that due to Major Siacci, of the Italian Artillery, and the foundation for the analytical work has been taken from a translation of Siacci's publication by Professor W. W. Johnson, U. S. Naval Academy.

The Ballistic Tables, computed by Lieut. Commander Ingersoll, are founded on Mayevski's discussion of Krupp's experiments at Meppen in 1881. The laws for the resistance of the air as deduced by Mayevski were published in the *Revue d'Artillerie*, April, 1883, and appear in this book converted to English units of weight and length. These equations are also to be found in "Exterior Ballistics," Captain James Ingalls, First U. S. Art., page 29.

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The projectiles used in Krupp's experiments were of modern construction, three calibres long, with ogival heads of two calibres radius, and fitted with a forced rotating band, giving smoothness of flight. These conditions corresponding very closely to modern practice, it is thought that a set of tables founded on such experience will give accurate results when the data for similar projectiles are used. These tables may be used, however, when other conditions obtain, by using proper ballistic coefficients, as will be explained in the work.

Numerous examples, taken mainly from Examination papers since 1887, have been added.

The notes prepared from time to time in the Department of Ordnance and Gunnery have been embodied when applicable.

NAVAL ACADEMY,
DEP'T OF ORDNANCE AND GUNNERY,
July, 1893.

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BALLISTIC TABLES FOR OGIVAL-HEADED PROJECTILES OF TWO CALIBRES RADIUS, LENGTH THREE CALIBRES, FORCED BAND.

CHAPTER I.

Definitions.—*Ballistics* is the science of the motion of projectiles, and *Exterior Ballistics* is that branch of the science which is concerned with their motion when outside the gun.

As certain terms will be continually used in the discussion of this subject, their definitions are here given.

The *line of sight* is a straight line passing through the two sight points; in the act of firing, it also includes the target.

The *line of departure* is the line in which the projectile is moving when it leaves the gun; it is, therefore, the tangent at the muzzle of the gun to the curve described by the projectile.

The *axis of the bore* is its geometrical axis, and is the line along which the centre of mass of the projectile should move while in the bore.

The *axis of the trunnions* is their common geometrical axis, and in recent guns it intersects the axis of the bore at right angles.

The *angle of elevation* is the angle included between the line of sight and the axis of the bore.

The *angle of jump* is the vertical angle which the axis of the bore describes in the act of firing, and is due to want of perfect rigidity of gun and carriage.

The *angle of sight* is the angle included between a line passing through the gun and target and the horizontal plane.

The *angle of departure* is the algebraic sum of the angles of elevation, jump and sight, or is the angle included between the line of departure and the horizontal plane.

The *angle of fall* is the angle which a tangent to the trajectory at the first point of impact makes with the horizontal plane.

The *trajectory* is the curve described by the projectile in passing from the muzzle of the piece to the first-point of impact. It is a curve of double curvature in the case of rifled guns, but is usually treated as a plane curve.

The *range* is the distance measured in a straight line from the muzzle of the piece to the intersection of the trajectory with the line of sight.

The *drift* is the constant deflection of a projectile from the plane of departure (the vertical plane through the line of departure), due to the rotation imparted by the rifling of the piece.

The *muzzle velocity* or *initial velocity* of a projectile is its velocity on leaving the piece, and is usually denoted by the abbreviation "I. V."

Muzzle velocity is always measured in feet per second, abbreviated thus—"f. s."

The *remaining velocity* of a projectile is its velocity at any given point of its trajectory. The remaining velocity must, in all cases, be less than the muzzle velocity, owing to the resistance of the air.

The *striking velocity* of a projectile is the velocity which it has on impact.

The *horizontal velocity* of a projectile is its velocity at any point resolved in a horizontal direction.

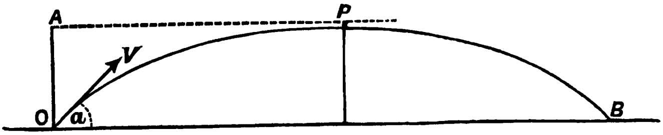
The *vertical velocity* of a projectile is its velocity at any point resolved in a vertical direction.

Thus, if V be the velocity of a projectile moving at an angle α to the horizon,

$$\text{Horizontal velocity} = V \cos \alpha,$$

$$\text{Vertical velocity} = V \sin \alpha.$$

Gravity is that force (caused by the attraction of the earth) which continually decreases the vertical velocity of the projectile in its ascending path and then brings the projectile to *rest* in a *vertical* direction. Gravity then causes it to descend, and continually increases its descending vertical velocity till it strikes the ground.



Thus if OV be the line of departure, α the angle of departure, and V the muzzle velocity of a projectile, its ascending vertical velocity will be

$$V \sin \alpha;$$

or if $V=650$ f. s., and $\alpha=30$,

$$\text{ascending vertical velocity} = 650 \times \tfrac{1}{2} = 325 \text{ f. s.}$$

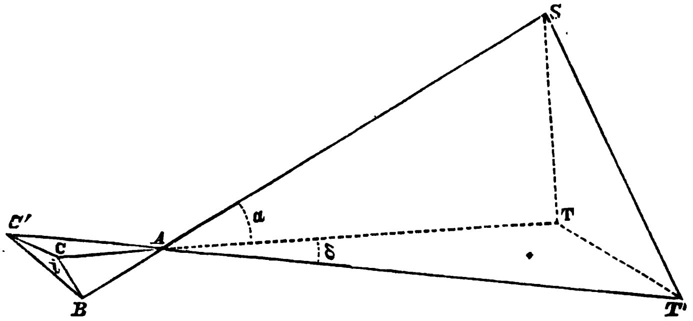
Gravity, and retardation due to the resistance of the air, combine to bring the projectile to rest at some point, *A*, if it were projected vertically upwards; but owing to the horizontal velocity of the projectile, it will have moved on at the same time to the point *P*, which is called the vertex, or *maximum height*, of the trajectory. Then the projectile commences to fall, and acquires a *descending vertical velocity* (under the influence of gravity *less* the retardation due to the resistance of the air), which *gradually increases* till the projectile strikes the ground. The horizontal velocity meanwhile has carried the projectile to the point *B*.

The Theory of the Sight Bar.—This theory, a knowledge of which is most essential, will also be presented here. There are several methods of laying guns so that the projectile will hit any desired point; but in our navy one way is adopted to the almost entire exclusion of all others. It consists in so marking the sight bar that the distance of the target to be hit being known, the bar may be set at that distance; then, when the breech of the gun is raised or lowered so that the line of sight includes the target, the axis of the bore is set at the correct angle for the projectile to hit the target.

In all investigations concerning the sight bar, it is permissible to neglect the dimensions of the gun when compared with the range of the projectile and its deflection (the distance it goes to the right or left); the results thus reached will, for practical purposes, be correct. For example: we may consider that the sights when the bar is down are in the axis of the bore, and may consider the range to be measured from the front, or rear sight, indifferently.

If a sight bar be correctly marked to hit a point at some assigned distance from the gun, it is evident that this mark will not enable us to hit whatever be the vertical position of the point; in other words, we cannot hit any desired point of a vertical circle described about the gun with the range as radius; for the present, then, the position of the point to be hit will be restricted to the horizontal plane through the gun.

When a sight bar is to be marked, the given quantities are the distance between sights, *AB*; the angle of elevation, *SAT* or *CAB*; the range, *AT*; and the deflection, *TT'*. In the figure let *BAS* represent the axis of the bore prolonged, and *CAT*, in the same vertical plane, the line of sight, A and C being the two sight points, and CB or $C'B$ the sight bar. Suppose that the projectile falls at



T', having been deviated from the plane ATS by TT'; draw $T'AC'$, and from the point C draw CC' at right angles to the plane ATS; then, if $C'AT'$ had been the line of sight, the point seen would have been the point struck. C' is consequently the proper position for the sight notch.

The distance TT' is the drift, for the range AT, due to the rifling; and if the drift was proportional to the range it would only be necessary to set the sight notch at a fixed distance towards C' from C to correct the drift for any range, the muzzle velocity being constant; but the drift is not proportional to the range, as has been found by trial. The sight bar in most guns is, therefore, placed in the rear sight box at an angle with the vertical plane through the two sight points, the angle i in the figure, and when the bar is set for a given range the sight notch C' moves to such a distance from C as will correct the drift for the given range.

The angle i is called the *permanent angle*. In the figure let $TT'=d$, $AT=R$, $CC'=k$, $BC=h$, $BC'=H$, $AB=l$, $CAB=\alpha=SAT$. The distance l is called the *sight radius*.

Then, since in most guns the three faces which we see of the tetrahedron $ABCC'$ are right angled at B and C, we have

$$\tan \alpha=\frac{BC}{BA}=\frac{h}{l} \therefore h=l \tan \alpha, \tag{1}$$

$$\tan i=\frac{CC'}{BC}=\frac{k}{h} \therefore k=h \tan i=l \tan \alpha \tan i, \tag{2}$$

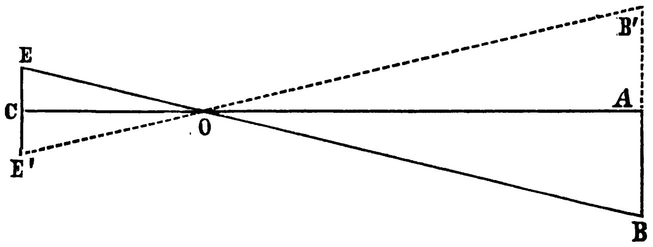
$$\tan\delta=\frac{CC'}{CA}=\frac{k}{l\sec\alpha}=\frac{d}{R},$$

$$\tan i=\frac{\tan\delta}{\sin\alpha}=\frac{d}{R}\operatorname{cosec}\alpha, \tag{3}$$

$$H=h\sec i=l\tan\alpha\sec i. \tag{4}$$

To recollect the above figure, it is convenient to remark that it consists of two triangular pyramids with their vertices coincident, formed by the lines *CT*, *C'T'*, *BS*, and that the planes of their bases when prolonged downward will intersect at the angle α.

The deviations of the projectile to the right or left of the plane of fire, due to motion of the gun and target and to the effect of the wind, are compensated by a sliding leaf attached to the head of the rear sight bar, which can be moved to the right or left, and which contains the rear sight notch. The method of computing the amount of the lateral deviations due to the influence of the wind and to the motion of gun and target will be explained in a subsequent chapter, but if the lateral deviation due to these causes combined is given, evidently by similar triangles, the range being known, the distance the rear sight notch must be moved laterally to compensate for the given amount of lateral deviation can be determined. Thus, in the following figure, if *AB* is the lateral deviation from the point *A*, which it is desired to hit, due to the combined action of the wind and to the motion of the gun and target; and if with the sight notch at *C* the gun is pointed at *A*, the projectile will fall at *B*; if, however *CE* had been computed from the similar triangles *EOC* and *OAB*, the sight notch moved to the point *E*, and the gun trained so that the new line of sight *EOB* coincides with *COA*, then the axis of the bore will project in the line *E'OB'*, and the projectile will



fall at the point A, as desired. *The general rule for setting the rear notch to compensate for lateral deviations is to move the notch in the direction it is desired the shot should go.*

EXAMPLES.

1. The range of an 8-inch M. L. R. projectile, with an I. V. of 1450 f. s. on a horizontal plane through the gun, for an angle of departure of $3^\circ\ 44'$, is 2000 yards. The jump angle is $+\ 28'$, and the drift or deviation to the right due to right-handed rifling is $3\frac{2}{3}$ yards. The distance between sights being $44.5''$, what is the permanent angle for the sight bar to correct for drift, and what distance should be set off on the bar for the 2000 yards mark?

Ans. $i = 1^\circ\ 50'\ 30''$.
$2.54''$.

2. With the present service charge, a projectile which leaves the 60-pdr. B. L. R. gun at an angle of departure of $2^\circ\ 42'$ will range 1000 yards on the horizontal plane through the gun. It is required to write down an expression which will fix the position of the 1000 yards mark on the sight bar of the gun, assigning letters to represent the values of the quantities which would ordinarily be required.

3. With an angle of departure of $2^\circ\ 42'$ the 60-pdr. B. L. R. projectile ranges 1000 yards on the horizontal plane through the gun in 2.72^s. The angle of jump is $+\ 27'$, radius of gun $39.33''$, and the rear sight is set at right angles to the axis of the bore and vertical; if this gun is to be fired at an object 1000 yards distant in its horizontal plane, while moving at a speed of 10 knots per hour in a direction at right angles to the line drawn from it to the target, and if the projectile retains the velocity of the gun during flight, determine the correct position of the rear sight notch, both laterally and vertically.

Ans. vert. $1.545''$.
hor. $0.5943''$.

4. An 8-inch rifled projectile leaves the gun at an angle of departure of 6°, and ranges 2300 yards on a horizontal plane through the gun. The radius of the gun is $44.5''$, jump $+\ 36'$, permanent angle $1^\circ\ 50'$ to the left. If, however, the sight bar was not set at a permanent angle, how much should the sliding leaf notch be moved horizontally to allow for the drift at the above range, and in which direction? What is the sight bar height for the above range?

Ans. $0.1346''$ to left.
$H = 4.2086''$.

5. The radius of a gun is 50 inches, and a projectile which leaves the gun at an angle of departure of 5°, the angle of sight being — 15′ and the jump 1°, falls at a distance of 1500 yards on a plane through the point of fall and the gun, and the deviation to the right due to rifling is 10 yards. What is the permanent angle for this gun and the sight bar height for 1500 yards when the bar is set at the permanent angle? *Ans.* $i = 5° 8′ 27″$.

$H = 3.734″$.

6. The angle of departure for the 60-pdr. M. L. R. for 1900 yards on a horizontal plane through the gun is 4° 27′, radius of gun 39.33″, angle of jump + 27′, permanent angle 1° 24′ to the left. What is the distance on the sight bar of the 1900 yards mark from the origin? What is the drift at this range?

Ans. Drift = 3.24 yards.

Distance = 2.753″.

7. An 8″ shell, weight 250 lbs., is fired with an initial velocity of 1700 f. s., and an angle of departure of 2° 12′. The jump is + 10′, the radius of the gun 81.44″ and the permanent angle 1° 50′. The range on the horizontal plane through the gun is 2000 yds. Find the height of the sight bar in inches.

Ans. $H = 2.8929″$.

8. The sight radius of a 6″ B. L. R. is 60.45″, weight of projectile 100 lbs., I. V. 2000 f. s., and the angle of departure for a horizontal range of 2000 yds. is 1° 41′. The jump angle is + 10′, the time of flight 3.5^s. Find the height at which the sight bar should be marked for the above range, and the distance and direction in which the sliding leaf should be moved in case the target and gun are moving in opposite directions, perpendicular to the line of fire, each with a speed of 15 knots (1 knot = 6000 ft.).

Ans. $H = 1.6006.″$

Leaf 1.763″ left.

9. The sliding leaf of the 6″ B. L. R., mark III, is marked in knots, right and left of the zero point, and the mark 20 is 1.17″ from zero. The radius of the gun is 66.81″. Your vessel and the enemy's are passing each other on opposite courses at a distance of 2000 yards, each vessel making 10 knots (1 k. = 6000 ft.). *How should the leaf be moved and set?* The gun being perfectly pointed at the water line abreast the mainmast, *where will the shot hit*? The time of flight for 2000 yds. is 3.5^s. *Ans.* 12′ abaft the mast.

10. The mean deviation of the 8″ B. L. R., I. V. 1700 f. s., at a range of 3000 yds., angle of departure 3° 35′, is 4 yds. right. The radius of the gun is 85.66″, jump 0. Determine the permanent angle to compensate this drift at the given range.

Ans. $i = 1° 13′ 20″$.

11. A 6″ shell is fired with an I. V. of 2000 f. s. The angle of departure is 2° 13′, the range 2500 yds., drift 3 yds. to the right, jump + 10′, and the radius of the gun is 60.543″. Determine the permanent angle to correct this drift and the height of the sight bar, when set as this angle, for the given range.

Ans. $i = 1° 55′$.
$H = 2.1683″$.

12. The rear sight of the 8″ B. L. R., mark III, is placed upon the gun in such a manner that when it is set at level and the sliding leaf at 0, the middle point of the sight notch is 0.2″ to the left of the vertical plane which passes through the front sight point and is parallel to the axis of the bore. In addition, the bar is inclined to the left by the permanent angle of 45′; the sight radius is 94.25″. Find what lateral deviation is compensated by this arrangement when the bar is set for a range of 2000 yds., the angle of elevation for which is 2° 15′. *Ans.* 5.269 yds.

13. To allow for the speed of the target, the sliding leaf of the 6″ B. L. R., mark III, is marked in knots. A vessel is passing at the rate of 22 knots (1 k. = 2000 yds.), at a distance of 2000 yds., for which the time of flight is 3.5^s. The mark 22 knots on the leaf is 1.3″ from zero and the distance between sights is 66.81″. If the gun is perfectly pointed at the enemy's mainmast how far forward or abaft will the shell strike? *Ans.* 4 yds. abaft.

14. The rear sight of the 6″ B. L. R., mark III, is placed upon the gun in such a manner that when it is set at level and the sliding leaf at 0, the sight notch is 0.187″ to the left of the vertical plane which passes through the front sight point and is parallel to the axis of the bore. In addition to being set to the left, the bar is inclined to the left by a permanent angle of 0° 54′; the distance between sights is 66.81″. Find what lateral deviation is compensated by this arrangement at a range of 2000 yds., the angle of elevation for which is 1° 48′ 10″. *Ans.* 19.755 feet.

15. A 6″ B. L. R., mounted 24 ft. above the water, is fired for range with an angle of departure of 1° 30′, and the shell strikes the water at a distance of 2000 yds. from the gun. The jump + 10′, radius of gun 65″, what is the vertical height of the sight bar for 2000 yds.? The drift being 10 yds. to the right and the rear sight vertical, what distance and in what direction must the sliding leaf be moved to correct for this drift?

Ans. 0.325″ to the left.

16. An 8″ shell, weight 250 lbs., is fired with an initial velocity of 1700 f. s. and an angle of departure of 2° 12′; the jump + 10′, radius of the gun 81.44″, and the permanent angle 1° 50′; the range on the horizontal plane through the gun is 2000 yds. Find the height of sight bar in inches. *Ans.* H; 2.893.

17. Given for the Hotchkiss 6-pdr. R. F. gun, range 2000 yds., angle of elevation 2° 46′, drift 3.37 yds. right, radius of gun 37.16″; determine the permanent angle to correct the given drift and the height of sight bar when fixed at this angle for the given range.

Ans. i; 2°.
H; 1.797″.

18. The rear sight of the 5″ B. L. R. is placed upon the gun in such a manner that when set at level, and the sliding leaf at 0, the rear sight notch is 0.14″ to the left of the vertical plane through the front sight point and parallel to the axis of the bore. The rear sight bar is also inclined to the left by the permanent angle of 45′. The sight radius of the gun is 48.75″. Find what lateral deviation is compensated by this arrangement for a range of 4500 yds., for which the angle of elevation is 5° 23′.

Ans. 18.39 yds.

The Determination of the Angle of Jump.—The vertical angle which the axis of the bore describes in the act of firing, due to straining and slackness of the gun and carriage, is called the *jump*. It may sometimes happen that there is also a horizontal angle of jump due to the same cause. The jump angles are carefully determined by experiment at proving grounds for each gun and carriage in the following manner: A screen of light material is put in front of the gun at a distance of from 15 to 50 yards, depending upon the size of the gun, far enough to be uninjured by the blast of discharge, and a point is marked on the screen in which the axis of the bore intersects it. The screen is usually of blotting paper, as it has been found that this material gives a clean hole the size of the cross section of the projectile. The point on the screen is determined by sighting through the bore by means of discs of wood or metal placed in the bore, the one at the breech with a pin-hole at its centre, and the one at the muzzle with cross wires.

If, then, x and y are the horizontal and vertical coordinates of the centre of the hole the shot makes in the screen referred to at this point, r the angle of jump, and d the distance of the screen from the muzzle, we have, approximately,

$$\tan r = \frac{y}{d} + \frac{gd}{2v^2};$$

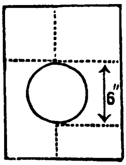
since $d = vt$ nearly, v being the initial velocity, and $s = \frac{1}{2} gt^2$.

The lateral jump angle is evidently $\tan^{-1} \frac{x}{d}$. A number of such determinations should be made, since the methods available are obviously inaccurate.

EXAMPLES.

1. A B. L. R, gun is laid level by a gunner's quadrant and fired at a screen distant 100 feet from the muzzle of the gun; the shot has a velocity of 1000 f. s.; the centre of the hole made by the shot is found by measurement to be 2 inches above the prolongation of the axis of the bore before firing. What is the *jump* of this gun?
Ans. 11′ 18″.

2. A 3-inch B. L. R. gun is sighted by means of a line of sight in the axis of the bore at a point on a vertical screen distant 50 feet from the muzzle. The gun being fired, I. V. = 1000 f. s., it is found that the point of the projectile hit the point aimed at. What is the *jump* of this gun? *Ans.* 2′ 45″.



3. To determine the jump of the 6-inch B. L. R., I. V. 2000 f. s., on the central pivot carriage, a screen made of heavy blotting paper was placed 100 ft. from the muzzle and the axis of the gun accurately pointed at the intersection of the dotted lines shown in the figure. The lowest point of the circumference of the shot hole was found to be 6″ below the horizontal line: find the jump. *Ans.* $j = -7' \; 12''$.

4. The 8-inch B. L. R., mark III, was sighted by cross wires in the bore and fired with I. V. 1700 f. s. at a point on a screen 100 feet from the muzzle; the jump was found to be 0, where with respect to the point aimed at did the point of the projectile strike?

Ans. 6.77″ below.

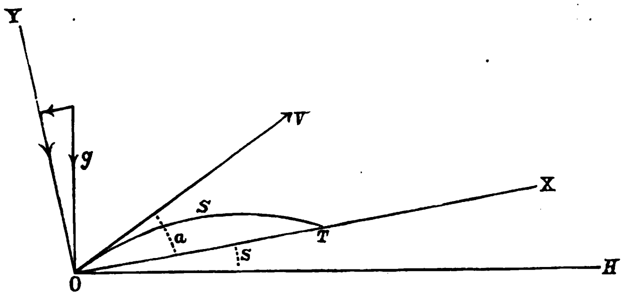
5. To determine the jump of the 6-inch B. L. R., I. V. 2000 f. s., the gun was fired at a screen 100 feet distant from the muzzle. The point of the shell pierced the screen 0.9″ above the point at which the axis of the bore was directed. Find the jump.

Ans. $j = 4'$.

CHAPTER II.

THE MOTION OF PROJECTILES IN A NON-RESISTING MEDIUM.

General Equation.—The resistance of the air to the motion of projectiles moving at ordinary velocities is so great that numerical results reached by considering their motion unresisted can seldom be applied in practice, but a correct understanding of the problems which arise is greatly aided by the facility acquired by the study of this comparatively simple case. It will in all cases be assumed that the only force acting upon the projectile during flight is that of gravity, and that this acts parallel to the vertical direction at the gun. The motion of the centre of mass of the projectile will therefore be confined to the vertical plane containing the gun and target.



Take the axis of X through the gun at O and target at T, and that of Y at right angles to it, as shown in the diagram ; let OH be the trace of the horizontal plane through the gun. Then, in the figure, OV is the direction of the axis of the bore, OT the line of sight, α the angle of projection, and s the angle of sight. The equations of motion are evidently, in the hypothesis assumed,

$$\left.\begin{aligned} \frac{d^2x}{dt^2} &= -g \cdot \sin s \\ \frac{d^2y}{dt^2} &= -g \cdot \cos s \end{aligned}\right\} \qquad (1)$$

Integrating, and determining the constants by assuming that the shot is projected with a velocity V in the direction making an angle a with the axis of X, and taking the origin of time at the origin of motion, we have

$$\left.\begin{aligned}\frac{dx}{dt} &= -gt.\sin s + V\cos a \\ \frac{dy}{dt} &= -gt.\cos s + V\sin a\end{aligned}\right\} \qquad (2)$$

Integrating again, we have, since x, y, and t are zero together,

$$\left.\begin{aligned}x &= Vt\cos a - \tfrac{1}{2}gt^2.\sin s \\ y &= Vt\sin a - \tfrac{1}{2}gt^2.\cos s\end{aligned}\right\} \qquad (3)$$

Multiplying the first and second of (3) by cos s and sin s respectively, and subtracting, we find for t,

$$t = \frac{x.\cos s - y.\sin s}{V\cos(a+s)}; \qquad (4)$$

and by substituting this value in the first of (3),

$$x = \cos a.\frac{x.\cos s - y.\sin s}{\cos(a+s)} - \frac{g.\sin s}{2}\left[\frac{x.\cos s - y.\sin s}{V\cos(a+s)}\right]^2. \qquad (5)$$

In this expression, putting $y = 0$, we find $x = 0$, and

$$x = \text{range} = \frac{2V^2}{g}.\frac{\sin a.\cos(a+s)}{\cos^2 s}. \qquad (6)$$

The equation to the curve described by the projectile, with the assumed origin and axes, is (5); it is evidently a parabola. The time of flight and range for any values of V, a, and s, may be found from (4) and (6). The formulæ are, however, greatly simplified by assuming that the gun and target are in the same horizontal plane; we have then, putting $s = 0$,

$$\left.\begin{aligned}x &= Vt.\cos a \\ y &= Vt.\sin a - \tfrac{1}{2}gt^2\end{aligned}\right\} \qquad (7)$$

whence the equation to the trajectory is

$$y = x.\tan a - \frac{gx^2}{2V^2\cos^2 a}. \qquad (8)$$

It is to be observed, however, that this is only a particular case.

Range.—Let R = range on the horizontal plane through the gun; then from (8),

$$R = \frac{V^2}{g}.\sin 2a; \qquad (9)$$

therefore, for any given value of V, the range is a maximum when $\alpha=45^\circ$. Equation (9) also shows that there are two values of α which are the complements of each other with which the point may be hit.

Time of Flight.—We also have from the first of (7), and obviously, since the velocity in X is constant when $s=0$,

$$T = \text{time of flight} \frac{R}{V.\cos\alpha}. \tag{10}$$

The Inclination of the Curve at any Point,

$$\frac{dy}{dx}=\tan\alpha-\frac{gx}{V^2.\cos^2\alpha};$$

by putting $\frac{dy}{dx}=0$ in this equation, we find the abscissa of the highest point of the curve; and by putting $x=R$, we find, calling β the angle between the tangent to the curve and the axis of X at that point, or the angle of fall,

$$\tan\beta=-\tan\alpha. \tag{11}$$

It is evident that the constants V and α may be so determined in (8) as to pass the trajectory through two points whose coordinates are given with reference to the gun. Let the points be (a_1, b_1) and (a_2, b_2): then we have from (8),

$$\left.\begin{aligned} b_1&=a_1\tan\alpha-\frac{g.a_1^2}{2V^2.\cos^2\alpha} \\ b_2&=a_2.\tan\alpha-\frac{ga_2^2}{2V^2.\cos^2\alpha} \end{aligned}\right\}$$

Therefore,

$$\left.\begin{aligned} \tan\alpha-\frac{b_1}{a_1}&=\frac{ga_1}{2V^2.\cos^2\alpha} \\ \tan\alpha-\frac{b_2}{a_2}&=\frac{ga_2}{2V^2\cos^2\alpha} \end{aligned}\right\}$$

Dividing these, member by member, and writing $\frac{b_1}{a_1}=\tan s_1$ and $\frac{b_2}{a_2}=\tan s_2$, we have

$$\tan\alpha=\frac{a_2.\tan s_1-a_1\tan s_2}{a_2-a_1}. \tag{12}$$

Also,

$$V=\frac{1}{\cos\alpha}\sqrt{\frac{g}{2}\cdot\frac{a_2-a_1}{\tan s_1-\tan s_2}}. \tag{13}$$

Ratio of Range on an Inclined Line to the Range on a Horizontal Line.—In the theorem just proved, it is to be noticed that the angle α is not the angle of elevation; and in order to investigate generally the subject of fire at objects which are not in the same horizontal plane with the gun, we shall resume equations (6) and (9); calling r the range on any inclined line: these are

$$r=\frac{2V^2}{g}\cdot\frac{\sin\alpha\,.\cos(\alpha+s)}{\cos^2 s},$$

and

$$R=\frac{2V^2}{g}\,.\sin\alpha\,.\cos\alpha.$$

From these, in which α is the angle of elevation as it is always measured in practice, we have

$$\frac{r}{R}=\frac{\cos(\alpha+s)}{\cos\alpha\,.\cos^2 s}. \tag{14}$$

By comparing (1) with the form to which it reduces when $s=0$, it is apparent that if, with a constant value of α, s increases slightly from 0, the range will diminish (since there will be a small retardation along X); as, however, while $(g \sin s)$ increases, $(g \cos s)$ diminishes, the range may, and does, subsequently increase and pass through the value it had when $s=0$. For negative values of s, (1) shows, when properly changed, that we shall always shoot over. In (14), let $\alpha=1°$ and $s=$successively 1°, 2°, 2¼°; we find

$$\frac{r}{R}=.99999,\ 1.0000,\ 1.0001.$$

From (14),

$$\frac{r}{R}=\sec s\,(1-\tan\alpha\,.\tan s).$$

Put

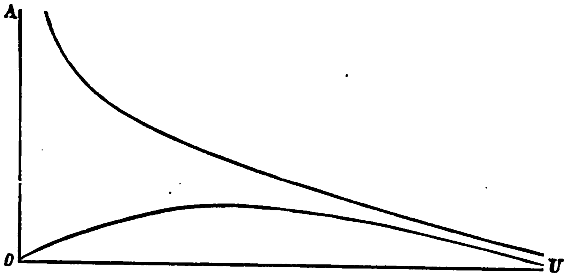
$$\frac{r}{R}=1,\ \tan s=u,\ \tan\alpha=a;\ \text{then}$$

$$1=(1+u^2)^{\frac{1}{2}}(1-au). \tag{15}$$

This is an equation of condition, which must be satisfied in order that $r=R$.

To trace its locus, square, by which we introduce the roots of $r = -R$; then we have

$$a^2u^2 - 2au^2 + a^2u + u - 2a = 0. \qquad (16)$$



The locus is of the form shown: it passes through the origin at an inclination $\tan^{-1}\frac{1}{2}$, and the maximum point is given by $u = \pm 1.27202$ and $a = \pm .30028$. The similar branches in the third quadrant are not shown.

If now we assign any value to u ($\tan s$), it is evident that there is one value of a for which $r = R$. The upper branch of the curve will be found to correspond with those values for which $r = -R$. It is also apparent, from the inclination of the curve at the origin, that for very small angles $r = R$, when $2a = u$ or $2\alpha = s$. Conversely, for any assigned value of a ($\tan \alpha$) less than .300282 (or $\alpha = 16° \; 42' \; 50''$), there are two inclinations of the plane for which $r = R$.

For example, if $a = .10$, whence $\alpha = 5.7$ about;
$u = .216$, 8.880, 10.911 about;
whence $\beta = 12.2°$, 83.5°, 84.8° "

By adding the last to 5.7°, we exceed 90°; this root, therefore, gives $r = -R$.

The Rigidity of the Trajectory.—From (14) it follows that, for small angles, the error resulting from using on an incline a sight bar which has been marked for use on a horizontal plane is not great, and in practice the assumption is frequently made that for small angles the range on any line is independent of its inclination to the horizontal. This is called the assumption of the Rigidity of the Trajectory, and consists, evidently, in assuming that the gun, trajectory, and line of sight—a chord—may be rotated through a vertical angle without change of form. To insert this assumption analytically, we have merely to suppose, in equation (8), that the axes and trajectory may be turned through a small vertical angle without change of form. Of course this assumption violates the hypothesis upon which that equation rests. The assumption that the trajectory is rigid so nearly represents the truth when the angles are small, and is so convenient, that it is always adopted in practice, and although the conclusion is deduced from the motion of projectiles in a non-resisting medium, the assumption will hold good within reasonable limits in the case of projectiles moving in a resisting medium.

The trajectory is assumed rigid in practice whenever a sight bar which has been marked for a gun at a certain height above the water is used when the height of the gun is changed, and also when the same sight bar is used to point at objects differing in vertical position.

EXAMPLES.

1. A gun is pointed at an angle of elevation of 1° 16′ 31″ above a line whose inclination is 45°; the muzzle velocity being 150 f. s., show that the time of flight is 2.93^s and the range on the incline 1434.2 yards.

2. A shot being projected at the same velocity as in Example 1, and at an angle of elevation of 1° 16′ 31″ above a horizontal line; show that the time of flight is 0.207^s and the range on this line 1036 yards.

3. The horizontal range of a projectile is 1000 ft. and the time of flight is 15 seconds. Required the angle of elevation, velocity of projection, and greatest altitude. *Ans.* $a = 74° \; 34'$.
$v = 250.29$ ft.
$H = 905.29$ ft.

4. Find the velocity and angle of elevation of a ball that it may be 100 ft. above the ground at the distance of one quarter of a mile, and may strike the ground at the distance of one mile.
Ans. $a = 5° \; 46' \; 05''$.
$v = 921.566$ ft.

5. What must be the angle of elevation of a body in order that the horizontal range may be equal to three times the greatest altitude? What, that the range may be equal to the altitude?

6. A body is projected at an angle of departure of 60°, with a velocity of 150 f. s.; find the coordinates of its position, the direction of its motion, and velocity at the end of 5 seconds.

Ans. $X = 375$.
$Y = 247.4'$.
$\beta = 157° 35'$.
$V = 81.19$ f. s.

7. A body is projected from the top of a tower 200 ft. high, at an angle of departure of 60°, with a velocity of 50 ft.; find the range on the horizontal plane passing through the foot of the tower, and the time of flight. *Ans.* $R = 128'$.
$T = 5.12^s$.

8. At what elevation must a shot be fired with a velocity of 400 ft. that it may range 2500 *yards* on a plane which *descends* at an angle of 30°? *Ans.* 72° 20'.

9. A 12-inch R. mortar shell, weighing 610 lbs., was fired at the Army O. P. G. at Sandy Hook, with an I. V. of 591 f. s. and an angle of departure of 73°. The observed range was 1939 yds. and the time of flight 36^s. Show what approximation can be made to these observed results by neglecting the resistance of the air; what is the greatest altitude of the projectile. *Ans.* $R = 2022$ yds.
$T = 35.1^s$.
$h = 4960$ ft.

10. What is the maximum height that could be attained by the projectile as described in the preceding example, with the given velocity, the resistance of the air being neglected, and at what elevation? *Ans.* 90°.
5424 ft.

Find the maximum range *in vacuo* of a projectile, having an initial velocity of 322 f. s., the ordinate of the vertex of the trajectory, and the time of flight. Use natural functions of α and give results in feet. *Ans.* $R = 3220$ ft.
$T = 10\sqrt{2}$ ft.
$Y = 805$ ft.

CHAPTER III.

THE RESISTANCE OF THE AIR.

Early Experiments.—The resistance of the air to projectiles moving in it with high velocities has long been known to be very considerable, and many attempts have been made to find a *law* which would account for the results obtained in practice.

Robins' experiments in 1742 are the first recorded attempts to obtain such a law, and were made by firing spherical bullets of lead from a small-arm and using a ballistic pendulum.

The following general statements were deduced:

(1) That for a velocity of the projectile not exceeding 1100 f. s., the resistance of the air varied as the *square of the velocity.*

(2) That at the velocity of 1100 f. s. (nearly the same as that with which sound is propagated through the air), this law of the resistance *changed.*

(3) That if the velocity exceeds 1100 f. s., then the absolute quantity of that resistance for the greater velocities was nearly three times as great as it should be by a comparison with the lower velocities.*

Hutton in 1790 made experiments with guns firing large spherical iron projectiles, and an improved ballistic pendulum, and from them deduced the law that if q denote the resistance of the air, and v the velocity of the projectile,

$$q \propto av + bv^2,$$

where a and b were constants determined by experiment.

General Didion, of the French army, in 1840 conducted a series of experiments at Metz, and proposed the following, which also consisted of two terms, the notation being the same as in the preceding,

$$q \propto .027\ (v^2 + .0023\ v^3).$$

This formula not having proved entirely satisfactory, a further series of experiments was made at Metz, in 1857, by Captain Welter (Professor in the School of Practical Artillery and Engineering), using an *electro-ballistic* pendulum. His conclusion was that the resistance of the air to *spherical* projectiles moving with high velocities was simply proportional to the *cube of the velocity;* or, $q \propto v^3$.

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* Robins' "Tracts on Gunnery," by Hutton, p. 181.

In 1860–1, at Gavre, experiments were made with *ogival-headed elongated* projectiles, by Prof Hélie, of the French School of Naval Gunnery, from which he came to the conclusion that the resistance of the air was proportional to the cube of the velocity, at least as long as the axis of the projectile did not deviate much from the tangent to the trajectory described by the centre of mass.*

Bashforth's Experiments.—From 1865 to 1870, and again in 1878–80, series of most valuable experiments were carried on in England by Prof. Bashforth, with spherical and ogival-headed projectiles, by means of his clock chronograph. The results showed that the resistance varies as follows:

For velocities between 900 and 1100 f. s. . . $q \propto v^6$, approximately.
" " " 1100 " 1350 " . . $q \propto v^3$ "
" " above 1350 f. s. . . . . $q \propto v^2$ "

It was also found that the resistance of the air *for the same velocity* and the same *form* of projectile, but of different calibres, varies exactly as the *square of the diameter* of the projectile. If the projectile presents a pointed head toward the opposing air, the resistance is less than if it is flat; according to Bashforth's experiments we may say, that if the resistance to a hemispherical head be taken as unity, the value of the resistance for different shaped heads, the projectiles having the same diameter and weight and the same velocity, is as follows:

1. Hemispherical head, ...	1.
2. Hemispheroidal head, ...	0.7848
3. Ogival head, radius 1 calibre, . . .	0.8253
4. Ogival head, radius 2 calibres, . . .	0.7840
5. Flat head, ...	1.5122

The ogival-headed projectiles used by Prof. Bashforth were one and one-half calibres radius, however, and in his earlier experiments studded projectiles were used.

Variation in the density of the air causes variation in the resistance, and it is assumed that it varies directly as the density.

Numerous formulas have been proposed to express the law of the resistance of the air, but it has been found to be very difficult to represent it as a simple continuous function which will admit of integration, hence it becomes convenient to assume that the resistance of the air varies as some power or function $f(v)$ of the velocity; the form of the function changing between certain limits of the velocity as found by experiment. A coefficient A is also introduced which is constant between the assumed limits and depends upon the *form* of the projectile.

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* "Traité de Balistique Experimentale," Hélie, 1865.

The Resistance.—From what has before been stated we may conclude, then, that the resistance of the air may be expressed in mathematical symbols by

$$q = \frac{d^2}{g} A f(v) \text{ in lbs. avoirdupois,} \tag{1}$$

in which d is the diameter of the projectile in inches, and v the velocity in feet per second; A is a very small decimal, usually; g appears in the denominator in order to reduce "absolute" to ordinary gravitation units.

The Retardation.—The *effect* of the resistance of the air is to cause retardation in velocity, and the amount of retardation is known from the elementary law in dynamics,

$$-\frac{w}{g} r = q, \text{ whence } r = -\frac{gq}{w}, \tag{2}$$

$\frac{w}{g}$ being the mass of the projectile. Combining (1) and (2) we have

$$r = -\frac{d^2}{w} A f(v), \tag{3}$$

which is the rate of the loss of velocity in feet per second caused by the resistance q in pounds.

From (1) it is seen that the *resistance* of the air is independent of the *weight* of the projectile; but the *retardation* is inversely as the weight; hence with projectiles of the same diameter but of different weights, the resistances are equal for the same initial velocity, but the retardation is less for the heavier one.

Mayevski's Equations for the Retardation.*—In 1881 extensive experiments were made by Krupp, at Meppen, with projectiles of varied calibre, but of nearly the same form—viz., three calibres in length, and with ogival heads of two calibres radius. These projectiles were also fitted with forced rotating bands, insuring smoothness and steadiness of flight. General Mayevski published in 1883* his deductions from Krupp's experiments, and which, since they are founded on experience with modern projectiles, will be used as the standard in this volume.

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* Revue d'Artillerie, April, 1883.

They are, reduced to English units of length and weight, as follows, for a standard density of the air, in which r = retardation or negative acceleration:

$$2800 \text{ ft.} > v > 1375 \text{ ft.}:$$

$$r = \frac{d^2}{w} A v^2; \ \log A = 6.1192437 - 10.$$

$$1375 \text{ ft.} > v > 1230 \text{ ft.}:$$

$$r = \frac{d^2}{w} A_1 v^3; \ \log A_1 = 2.9808825 - 10.$$

$$1230 > v > 970 \text{ ft.}:$$

$$r = \frac{d^2}{w} A_2 v^5; \ \log A_2 = 6.8018436 - 20.$$

$$970 \text{ ft.} > v > 790 \text{ ft.}:$$

$$r = \frac{d^2}{w} A_3 v^3; \ \log A_3 = 2.7734232 - 10.$$

$$790 \text{ ft.} > v > 0 \text{ ft.}:$$

$$r = \frac{d^2}{w} A_4 v^2; \ \log A_4 = 5.6698755 - 10.$$

To Determine the Resistance Experimentally.—If a projectile be fired from a gun, with the axis of the bore set so that the path of the projectile as it leaves the muzzle will be horizontal, for a short distance the path described by the projectile may be considered, without material error, a horizontal right line. The loss of velocity over this short distance may then be considered as due alone to the resistance of the air, since the only other force, gravity, which acts on the projectile, may be neglected, as its action is at right angles to the path of the projectile.

Knowing the velocities v_1 and v_2 at two points of the same trajectory, measured by chronograph, at a distance l apart, the *mean* resistance q for this short portion of the path of the projectile is determined from the formula

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* Revue d'Artillerie, April, 1883.

$$q = \frac{w}{2gl}(V_1^2 - V_2^2), \qquad (4)$$

$$ql = \text{loss of energy},$$

$\frac{wv^2}{2g}$ being the energy of the projectile when the velocity is v f. s., w being the weight in pounds, and g the acceleration of gravity.

This resistance can be considered as corresponding to a velocity intermediate between v_1 and v_2, say to their arithmetical mean, or $\frac{v_1 + v_2}{2} = V$; this value substituted in the expression for q, formula (4), gives

$$q = \frac{wV}{gl}(V_1 - V_2). \qquad (5)$$

By suitable variation of the charge, v_1 and v_2 are made to vary; and from the values of q thus determined, a *law* for the resistance can be deduced. It is evident, however, from the formula that no account is yet taken of the variable density of the air, and it must be distinctly kept in mind that these values of the resistance can only be compared among themselves so long as the density of the air remains the same, and, further, so long as the projectiles are of the same calibre and form. When the experiments are not carried out under these conditions, it is necessary to refer the quantity q to a standard density and projectile. If δ is the density of the air at the time the experiment is made, and δ_1 is the standard density adopted, the second member of equation (5) must be multiplied by $\frac{\delta_1}{\delta}$. It is usual, in comparing results with projectiles of different calibres, to reduce the resistance obtained to that per unit (square foot) of cross section. Equation (5) then reduces to the following:

$$q = \frac{wv\delta_1}{gl\pi R^2\delta}(V_1 - V_2),$$

R being the radius of cross section in feet.

EXAMPLES.

1. A 6-inch projectile, ogival head, weight 70 pounds, is fired through screens, and the velocity is determined at two points 600

feet apart by chronograph, as follows: $V_1 = 1595$ f. s., $V_2 = 1866$ f. s.; what is the resistance in pounds? *Ans.* 644 pounds.

2. A 6-inch ogival-headed projectile, weight 70 pounds, is fired through screens 450 feet apart, and the velocity at the first screen is found to be 1846 f. s., at the second 1790 f. s. A six-inch flat-headed projectile, weight 70 pounds, is fired through the same screens, and the velocity at the first screen is found to be 1929 f. s., and at the second 1790 f. s. Compare the resistances for these different forms of head.

$$Ans.\ \frac{\text{Flat head}}{\text{Ogival head}} = \frac{q_1}{q_2} = 2.5388.$$

3. A 12-inch ogival headed projectile, weight 850 pounds, is fired through screens, and the velocity at two points 500 feet apart is found by chronograph to be 2000 f. s. and 1955 f. s. respectively. The weight of a cubic foot of air at the time of firing is found to be .950 as compared to the standard, the weight of which is unity. What is the resistance per unit (square foot) of cross section for this projectile under standard conditions of the air?

Ans. 6296.5 pounds per square foot.

4. Find the resistance of the air and the retardation of a projectile in which $d = 12.5$ inches and $w = 802.25$ pounds, when moving in air with a velocity of 1400 f. s.

Resistance $= 1251.6$ pounds.
Retardation $= 50.24$ f. s.

5. Find the resistance of the air to a base-ball of three inches diameter when moving at a velocity of 100 f. s., assuming the resistance to the sphere and standard ogival to be in the ratio 1 : .8.

Resistance $= .1634$ pound.

NOTE.—To solve examples 4 and 5, it is necessary to know the form of $f(v)$ in equations (2) and (3); as will be seen later, the form of the function for the velocites given is v^2, and the value of A for 4 is $\log A = 6.11924 - 10$, the value of A for 5 being $5.66987 - 10$.

CHAPTER IV.

THE COMPUTATION AND USE OF BALLISTIC TABLES.

Time and Velocity.—If r denote the retardation of a projectile moving in air in standard conditions, we have, equation (3), Chapter III,

$$r = -\frac{d^2}{w} A f(v).$$

If we know the form of $f(v)$ and the value of A, we may find the retardation, and consequently the force resisting any projectile.

We have also

$$r = \frac{dv}{dt} = -\frac{d^2}{w} A f(v). \tag{1}$$

Space and Velocity.—From equation (1), multiplying numerator and denominator of the first member by ds, we have, since ds is the differential of the distance passed over, and dt of the time,

$$\frac{ds\,dv}{dt\,ds} = \frac{v\,dv}{ds} = -\frac{d^2}{w} A f(v). \tag{2}$$

Equations (1) and (2) are of great importance; and, since they give a relation between time and velocity, and one between distance passed over and velocity, they would furnish the complete solution of ballistic problems, in which we ordinarily require relations between time of flight, range, and velocity, if gravity did not act, and if we know the form of $f(v)$ and the value of A. The form of $f(v)$ is not the same, however, for all velocities, but experiment has shown that between certain limits of the velocity it may be considered as approximately true that the retardation varies according to a certain power of the velocity, and that the value of A may be considered as constant between those limits. For example, for velocities between 3600 feet and 1375 feet, the retardation varies as v^2; between 1375 feet and 1230 feet, as v^3; between 1230 feet and 970 feet, as v^6, and so on, as shown in Mayevski's equations for the retardation (see Chapter III). Evidently, by substituting the proper form of $f(v)$ and the corresponding value of A, and integrating over narrow limits for each equation, tables connecting time and velocity and space and velocity may be constructed. This plan has been followed out in the case of Table I, and the columns headed S_v and T_v have thus been computed.

Since the value of the factor $\frac{d^2}{w}$ is different in different guns, it is more convenient to make out the tables for a projectile in which $\frac{d^2}{w}=1$; and the tables are so made out. Also, in *all* the tables given, d, the diameter of the projectile, must be taken in inches; w, the weight of the projectile, in pounds; the velocities, in feet per second; the distances passed over, in feet, and the times in seconds.

Details of the Computation.—From equation (1) we have

$$dt=-\frac{wdv}{d^2Af(v)}; \text{ or when } \frac{d^2}{w}=1,$$

$$dt=-\frac{dv}{Af(v)}. \qquad (3)$$

When we substitute in (3) for $f(v)$ the form as given by Mayevski, Chapter III, and the corresponding value of A, we get as follows: *

$$3600 \text{ ft.} > v > 1375 \text{ ft.}:$$

$$dt=-\frac{dv}{Av^2}.$$

$$1375 \text{ ft.} > v > 1230 \text{ ft.}:$$

$$dt=-\frac{dv}{A_1v^3}.$$

$$1230 \text{ ft.} > v > 970 \text{ ft.}:$$

$$dt=-\frac{dv}{A_2v^6}.$$

$$970 \text{ ft.} > v > 790 \text{ ft.}:$$

$$dt=-\frac{dv}{A_3v^3}.$$

$$790 \text{ ft.} > v > 500 \text{ ft.}:$$

$$dt=-\frac{dv}{A_4v^2}.$$

Integrating the first of this set of equations between the limits $v_0=3600$ ft. and v, and making $t=0$ when $v=3600$ ft.; calling T_v the value of t when the velocity $=v$ feet: we get the following:

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*We assume that above 2300 f. s. the resistance varies as Av^2.

$$\int_0^t dt = -\int_{3600}^{v} \frac{dv}{Av^2} = \frac{1}{Av} - \frac{1}{3600A}\,; \text{ or}$$

$$T_v = \frac{1}{Av} + C;$$

and in a similar manner we get values for T_v for the other equations, so that they may be written as follows:

$$\left.\begin{array}{c}
3600 \text{ ft.} > v > 1375 \text{ ft.}: \\
T_v = \dfrac{1}{Av} + C. \\
1375 \text{ ft.} > v > 1230 \text{ ft.}: \\
T_v = \dfrac{1}{2A_1v^2} + C_1. \\
1230 \text{ ft.} > v > 970 \text{ ft.}: \\
T_v = \dfrac{1}{4A_2v^4} + C_2. \\
970 \text{ ft.} > v > 790 \text{ ft.}: \\
T_v = \dfrac{1}{2A_3v^2} + C_3. \\
790 \text{ ft.} > v > 500 \text{ ft.}: \\
T_v = \dfrac{1}{A_4v} + C_4.
\end{array}\right\} \qquad (4)$$

In equations (4) the values of A_1, A_2, A, etc., are obtainable from Mayevski's equations (see Chapter III). In the first of (4) the value of C may be so taken that the value of T_v will be zero when $v =$ 3600 feet. In the first case the value of C which will make $T_v = 0$ when $v = 3600$ feet, is -2.110832; log A for velocities between 3600 feet and 1375 feet is $6.1192437 - 10$; so that the first equation becomes

$$T_v = [3.8807563]\frac{1}{v} - 2.110832.$$

The number within brackets is the logarithm of the constant coefficient in the first term.

By this formula, $T_{1375} = 3.416.$

Proceeding in a similar manner, we get for the second equation of the set, for velocities between 1375 feet and 1230 feet, log $A_1 =$ $2.9808825 - 10$,

$$T_v = [6.7180875]\frac{1}{v^2} + C_1.$$

Now the value of C_1 must be so taken that there will be no abrupt change in the tables when $v = 1375$ feet or when the law of the retardation changes; hence we may obtain C_1 as follows:

$$T_v = [6.7180875]\frac{1}{(1375)^2} + C_1 = 3.416,$$

since both laws for the retardation must hold good for $v = 1375$ feet, and we find $C_1 = +.65204$; so the second equation becomes

$$T_v = [6.7180875]\frac{1}{v^2} + .65204.$$

Similarly we proceed with all the others, and the result may be tabulated as follows:

$$\left.\begin{array}{c}
3600 > v > 1375: \\
T_v = [3.8807563]\frac{1}{v} - 2.1108. \\
1375 > v > 1230: \\
T_v = [6.7180875]\frac{1}{v^2} + .65204. \\
1230 > v > 970: \\
T_v = [12.5960964]\frac{1}{v_4} + 2.38196. \\
970 > v > 790: \\
T_v = [6.9255468]\frac{1}{v^2} - 2.11510. \\
790 > v > 500: \\
T_v = [4.3301245]\frac{1}{v} - 15.68695.
\end{array}\right\} \qquad (5)$$

Proceeding in a similar manner with equation (2), we get the following expressions for computing the values of S_v:

$$\left.\begin{array}{c}
3600 > v > 1375: \\
S_v = 62225.7 - [4.2429714]\log v. \\
1375 > v > 1230: \\
S_v = [7.0191175]\frac{1}{v} - 291.46. \\
1230 > v > 970: \\
S_v = [12.7210351]\frac{1}{v^3} + 5382.54. \\
970 > v > 790: \\
S_v = [7.2265768]\frac{1}{v} - 6123.7. \\
790 > v > 500: \\
S_v = 157890.8 - [4.6923402]\log v.
\end{array}\right\} \qquad (6)$$

The values of T_v and S_v have thus been computed for properly chosen intervals of v for the value of $\frac{d^2}{w} = 1$. For any other value of $\frac{d^2}{w}$, evidently the tabular values must be multiplied by the particular value of that factor, or rather by its reciprocal $\frac{w}{d^2}$.

In a similar manner the values of other functions required in the solution of ballistic problems, and depending upon the velocity, may be computed and tabulated. The expressions for other functions will hereafter be given, but the manner of their computation will not be dwelt upon, as it presents no essential features different from that already given.

The Use of the Tables.—The use of the tables is best explained by examples. Suppose we wish to find in how long a time a 6-inch projectile whose weight is 100 pounds, and which is moving with a velocity of 2100 f. s., will have its velocity reduced to 1900 f. s. Entering Table I with 1900 feet as an argument, we find in the column T_v the value 1.889; and again, with 2100 f. s. as argument, we find the tabular value 1.508; and the difference of these values, or .381, will be the time in seconds it will take a unit projectile for which $\frac{d^2}{w} = 1$, to have its velocity reduced from 2100 f. s. to 1900 f. s.; but $\frac{d^2}{w}$ for the projectile in question $= \frac{36}{100}$; therefore the quantity 0.381^s should be multiplied by the reciprocal of $\frac{d^2}{w}$, and the result required is $0.381^s \times \frac{100}{36} = 1.583^s$. This leads to a convenient manner of writing equations for the solution of problems involving *time and velocity* and *space and velocity*, as follows:

$$\frac{d^2}{w} T = T_{v_2} - T_{v_1}, \tag{7}$$

$$\frac{d^2}{w} S = S_{v_2} - S_{v_1}, \tag{8}$$

T_v and S_v signifying the tabular numbers in Table I corresponding to the velocity V.

It is required to know the distance passed over by the projectile in question while its velocity is being reduced from 2100 f. s. to 1900 f. s.

We have, equation (8),

$$\frac{d^2}{w}S = S_{1900} - S_{2100},$$

$$S = \frac{100}{36}(4855.8 - 4095.5) = 2112.5 \text{ feet.}$$

Suppose now we wish to find the remaining velocity after a time T for a given projectile. Take a 6-inch projectile, weight 100 pounds, initial velocity 2000 f. s., and we wish to know the remaining velocity after 0.5^s. We have, equation (7),

$$\frac{36}{100}.\times 5 = T_v - T_{2000} = T_v - 1.689,$$

$$\therefore T_v = 1.689 + .18 = 1.869.$$

Entering Table I, in the column T_v we find opposite the quantity 1.868 in the column marked V, the velocity 1910 f. s. The tabular difference for T_v is 2, and the correction to be applied is evidently .5, so the velocity required is 1709.5 f. s.

Again, suppose we wish to find the remaining velocity for a 6-inch projectile, weight 100 pounds, after it has traversed a distance of 1000 feet, the initial velocity being 2000 f. s.: we have, equation (8),

$$\frac{36}{100} \times 1000 = S_v = S_{2000} = S_v - 4466.2,$$

$$\text{or, } S_v = 4466.2 + 360 = 4826.2.$$

Entering the column S_v, in Table I, with this quantity, we find the remaining velocity required to be 1907.5 f. s.

As there is no direct relation between *time* and *space*, when it is required to find the time a projectile takes to travel over a given space, or the reverse, it is necessary to know the initial or final velocity, and to use both equations (7) and (8); with one of them the initial and final velocities are determined for use in the other equation.

The other tabular values in Table I are taken out in a similar manner and special explanation is not necessary. Special regard should be given to the columns of differences if accurate results are required.

Limits of Accuracy.—The above methods furnish the solution of some of the simpler ballistic problems which occur: and it is evident that whenever the distance passed over enters, their application must be confined to those cases in which the range, a chord, is sensibly equal for practical purposes to the curved path the shot actually pursues. It is difficult to assign precise limits beyond which their use should not extend, as these limits depend upon the ranges considered, the velocity, the calibre, as well as upon other things of minor importance. For medium guns with high velocity, the limit should, perhaps, not exceed an angle of departure of 3°, which would correspond to a range of about 2500 yards.

Within these limits, (7) and (8) furnish the solution of all problems in so far as the time and one coordinate, the horizontal one, along which the ranges are measured, is concerned.

SIMPLE BALLISTIC PROBLEMS.

1. It is required to find the muzzle velocity of the 3-inch B. L. R. in which $d = 3$, $w = 7$; the projectile is fired through two screens, and its velocity at the middle point between the screens is found to be 1062 f. s., this point being seventy feet from the muzzle of the gun. *Ans.* 1069.44 f. s.

2. To find the time of flight with a projectile in which $w = 64$, $d = 6$, when projected with a velocity of 1457 f. s., for a range of 1000 *yards*. For this velocity and range the length of the curved path which the shot will describe may for practical purposes be taken equal to the length of its chord, which is the range stated. In a case of this nature we must first find the velocity at the end of the range by (8), and then (7) will evidently furnish the solution. $T = 2.297$.

3. Prove that the rate of loss of velocity of ogival projectiles is inversely proportional to their calibre if their form is *similar*; that is, if their heads are so formed that their weights are proportional to the cube of the calibre.

4. A projectile in which $d = 6$ inches, $w = 72$ pounds, is fired at an iron target 100 yards from it, with a muzzle velocity of 2000 f. s. Find the striking velocity and time of flight. Here the arc considered of the trajectory is sensibly equal to its chord, the range.

Striking velocity = 1961.1 f. s.
Time of flight = .15 sec.

5. The same projectile as in 4 is fired at an iron target fifty yards from it, and must have 2000 f. s. velocity to penetrate. Find muzzle velocity and time of flight. $V = 2020.$
$T = .076.$

6. The same projectile, that in 4, is fired from a gun which is 55 feet from a target screen, a second screen being 90 feet from the first. By means of a chronograph, the interval of time between the rupturing of the two screens is found to be 0.05 sec. Find the muzzle velocity. The velocity at the middle point between the two screens is, very approximately, $V_2 = \frac{90}{.05} = 1800$ f. s.
Muzzle velocity $= 1811.9.$

Corrections to be applied for Variations of Thermometer and Barometer, and for Projectiles differing from the Standard.—To determine the correction to be applied in the computation of trajectories for variations of thermometer and barometer.

The values in the ballistic tables have been reduced to a standard atmospheric condition of thermometer 62° F., barometer 30 inches. The standard projectile of the tables is three calibres in length with an ogival head of two calibres radius and a forced band. It corresponds to that of the new heavy guns for the U. S. navy.

Assuming the density of the air to be unity under the standard conditions, let v_0 be the volume of unit weight under these conditions, α the coefficient of expansion $= .002178$, B the observed height of the barometer, and t the difference between 62° and the observed temperature. The volumes varying inversely as the pressure, or barometric height, we have for the volume of unit weight under observed conditions $v_0 (1 \pm \alpha t) \frac{30}{B}$.

Let δ be the ratio of the density under observed conditions to the density under standard conditions; then, since the densities vary inversely as the volumes,

$$\delta = \frac{v_0}{v_0(1 \pm \alpha t)} \cdot \frac{B}{30} = \frac{B}{(1 \pm \alpha t)\ 30}.$$

The tables of values of δ, for the standard adopted, are usually printed in connection with ballistic tables.

It is *assumed* that the retardation varies directly as the density of the air, and, since it varies also directly as the square of the diameter, we may make δ the coefficient of d^2.

The ratio of the retardation of the projectile used to that of the standard projectile is called the *Coefficient of Form*. It is denoted by i and obviously will appear in the coefficient of d^2.

The quantity $\frac{w}{d^2}$, called the *Ballistic Coefficient*, and denoted by C, becomes generally $\frac{C}{\delta i}$.

The value of i, for any projectile, may be determined by measuring its velocity, by means of the chronoscope, under standard atmospheric conditions, at two points of the trajectory S feet apart. Taking $S_{v'}$ and $S_{v''}$ from the tables we have

$$\frac{i}{C}S = S_{v''} - S_{v'},$$

$$i = \frac{C(S_{v''} - S_{v'})}{S}.$$

To correct the observed range, approximately, for variations from the standard atmospheric conditions, if, for the same initial velocity and angle of departure, the vertical resistance of the air is assumed to be the same under the observed and under the standard atmospheric conditions, the maximum ordinates of the two trajectories, and also the times of flight will be equal. For the flat trajectories of high-powered guns the vertical component of the velocity is small, and this assumption can be made without material error.

Under the observed conditions let X be the range, V the initial velocity, and v the final velocity. Let X' and v' be the required range and final velocity under standard conditions.

The projectile being standard, we have

$$S_v = \frac{\delta}{C}X + S_V,$$

from which S_v and, by the tables, T_v are found.

To find the time of flight

$$T = \frac{C}{\delta}(T_v - T_V).$$

Assuming T to be the same under standard conditions,

$$T_{v'} = \frac{T}{C} + T_V,$$

from which $T_{v'}$ and, by the tables, $S_{v'}$ are found.

Finally, $$X' = C(S_{v'} - S_v).$$

It is to be noted that this correction can only be approximate, owing to the uncertainty as to the law of the variation of the resistance with the density.

The correction to be applied to the observed range for the effect of the wind is also uncertain, and therefore experimental firing should, if possible, be carried on in calm weather and under standard atmospheric conditions.

The ballistic formulas are to be habitually written with C in the place of $\frac{w}{d^2}$.

The following Table for $\frac{w}{d^2} = C$ for the different service projectiles will be found convenient:

VALUES OF C FOR SERVICE GUNS, U. S. NAVY.

Calibre inches.	Gun. Class.	W Lbs.	C Log.	C Co Log.	$\frac{C}{2}$ Log.
0.315	Small Arm.	.03394	9.53409–10	0.46591	9.23306–10
0.45	"	.0686	9.52990–10	0.47010	9.22887–10
1.46	R. F.	1.1	9.71268–10	0.28732	9.41165–10
1.85	"	3.3	9.98417–10	0.01583	9.68314–10
2.24	"	6.0	0.07765	9.92235–10	9.77662–10
3.0	Boat Gun.	7.0	9.89086–10	0.10914	9.58983–10
4.0	R. F.	33.	0.31439	9.68561–10	0.01336
5.0	"	50.	0.30103	9.69897–10	0.00000
5.0	B. L. R.	60.	0.38021	9.61979–10	0.07918
6.0	"	100.	0.44370	9.55630–10	0.14267
8.0	"	250.	0.59176	9.40824–10	0.29073
10.0	"	500.	0.69897	9.30103–10	0.39794
12.0	"	850.	0.77106	9.22894–10	0.47003
13.0	"	1100.	0.81350	9.18650–10	0.51247
16.0	"	2000.	0.89279	9.10721–10	0.59176

g, 32.2; log, 1.50786; co. log, 8.49214–10.
π, 3.1416; log, 0.49715; co. log, 9.50285–10.

To find Approximately the Ordinate of a point on the Trajectory and the Dangerous Space for a Target of given height above the horizontal plane through the Gun from the data given in the Range Table.—It is assumed, without material error, in computing the ordinates of the descending branch of the trajectory (*i. e., a flat trajectory*) that the chord *AP* is equal to the distance *AC*, and, entering the range table with the distance *AC* as a range $= AP$, the corresponding angle of elevation α' is taken out, and also α corresponding to *AB*; then

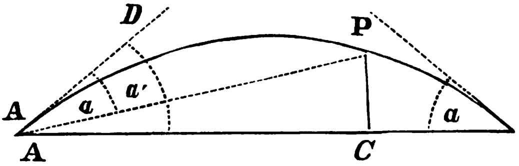

FIG. 1.

$$CP = AC \tan(\alpha' - \alpha); \quad DAP = \alpha; \quad DAC = \alpha'.$$

The target being at *C*, Fig. 1, this gives the vertical distance above the target of a shot falling at *B*.

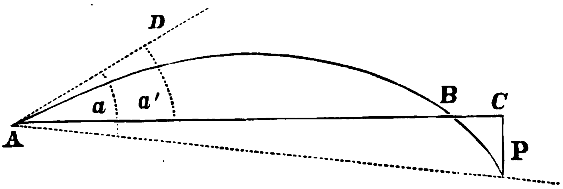

FIG. 2.

Similarly in Fig. 2, the target being at *C*, we have for the vertical distance under the target of a shot falling at *B*, $CP = AC \tan(\alpha - \alpha')$.*

To find the Ordinate of the Vertex.—The vertical component of the velocity in the case of a flat trajectory is so small *for ordinary ranges* that we may neglect the vertical resistance of the air and consider the vertical motion as affected by gravity only. The time of flight to the vertex will be $\frac{1}{2}T$, as *in vacuo*, and the ordinate may be found, approximately, by substituting $\frac{1}{2}T$ for t in the gravity formula, or

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* This method is due to Lieut. Haeseler, U. S. N.

$$Y=\frac{gT^2}{8}.$$

The Dangerous Space for an object of a given height, CP, Fig. 1, is the distance BC, from the point of fall to the foot of the ordinate, CP, which is equal in height to the target. ω being the angle of fall, a sufficient approximation, generally, is

$$BC=CP\cot\omega.$$

The following portions of range tables for the 5", 6" and 8" B. L. R. are given, from which examples illustrating the method of finding, approximately, the ordinate of a point on the trajectory and the dangerous space for an object of known height can be taken.

RANGE TABLE.—5" *B. L. from Practice.*

I. V., 1700 *f. s.*; *jump*, $+0°\ 3'\ 35''$; *radius of gun*, 48.75".

Range, yds.	α	T	ω	f. v.
1000	1° 00′	1.88^s	1° 10′	1466
1200	1 13	2.29	1 28	1402
1400	1 28	2.70	1 46	1359
1600	1 43	3.14	2 07	1318
1800	1 59	3.61	2 30	1278
2000	2 16	4.07	2 54	1239
2200	2 33	4.55	3 20	1207
2400	2 51	5.02	3 46	1177
2600	3 09	5.50	4 13	1150
2800	3 29	5.98	4 41	1126
3000	3 49	6.47	5 11	1103
3200	4 10	7.00	5 45	1082
3400	4 31	7.55	6 19	1062
3600	4 54	8.12	6 56	1042
3800	5 20	8.71	7 37	1024
4000	5 49	9.40	8 19	1006
4200	6 23	10.24	9 23	984
4400	7 04	11.16	10 29	963

RANGE TABLE.—8″ *B. L. R., from Practice.*

I. V., 1700 *f. s.*; *w*, 250 *lbs.*; *jump*, 0; *radius of gun*, 85.66.″

Range, yds.	α	T	ω	f. v.
1000	1° 04′	1.93^{s}	1° 08′	1531
1200	1 17	2.34	1 24	1499
1400	1 32	2.73	1 40	1469
1600	1 46	3.17	1 58	1439
1800	2 00	3.58	2 16	1411
2000	2 15	4.00	2 35	1383
2200	2 30	4.43	2 55	1357
2400	2 46	4.87	3 15	1331
2600	3 01	5.32	3 37	1306
2800	3 18	5.78	3 59	1282
3000	3 35	6.24	4 23	1258
3200	3 52.5	6.70	4 47	1236
3400	4 10	7.16	5 11	1216
3600	4 27	7.64	5 38	1197
3800	4 45.5	8.13	6 06	1179
4000	5 04	8.62	6 34	1161
4200	5 23	9.18	7 04	1145
4400	5 43	9.74	7 34	1129
4600	6 03	10.30	8 06	1114
4800	6 25	10.86	8 40	1100
6000	8 42	12.39	12 21	1028

RANGE TABLE.—6″ *B. L. R., Mark* 3.

I. V., 1700 *f. s.*; *w*, 100 *lbs.*; *jump*, —3′; *radius of gun*, 66.81″.

Range, yds.	α	T	ω	f. v.
1000	1° 10′ 10″	1.94^{s}	1° 12′	1455
1200	1 24 27	2.36	1 29	1409
1400	1 39 05	2.82	1 47	1365
1600	1 54 10	3.29	2 08	1323
1800	2 09 45	3.77	2 30	1284
2000	2 25 53	4.25	2 55	1246
2200	2 42 45	4.75	3 22	1210
2400	3 00 23	5.25	3 51	1177
2600	3 18 43	5.75	4 25	1145

RANGE TABLE.—6″ *B. L. R., Mark* 3—*Continued.*

I. V., 1700 *f. s.*; *w*, 100 *lbs.*; *jump*, —3′; *radius of gun*, 66.81″.

Range, yds.	α	T	ω	f. v.
2800	3° 37′ 45″	6.25^{s}	5° 01″	1115
3000	3 57 33	6.75	5 39	1087
3200	4 18 10	7.27	6 19	1062
3400	4 39 35	7.80	7 00	1041
3600	5 01 50	8.35	7 42	1023
3800	5 25 00	8.92	8 24	1007
4000	5 49 05	9.52	9 06	991
4200	6 14 10	10.13	9 50	975
4400	6 40 15	10.77	10 34	959
4600	7 07 20	11.44	11 21	945
4800	7 35 30	12.12	12 11	931
5000	8 04 45	12.80	13 01	917
5200	8 35 05	13.48	13 51	903

MISCELLANEOUS EXAMPLES.

1. What is the *approximate maximum height* of the trajectory of the 6-inch B. L. R. projectile for a range of 1000 yards, I. V. = 2000 f. s., $w = 100$ pounds, assuming that the projectile reaches the highest point in one-half the total time of flight? What *distance* from the gun is this point? *Ans.* $H = 10.45$ feet, $S = 1568$ feet.

2. An 8-inch ogival-headed projectile weighing 250 pounds, radius of head 2 calibres, is fired through two screens 100 feet apart and 100 and 200 feet from the gun respectively; the time between screens is $\frac{1}{20}$ of a second; what is the initial velocity of the above projectile, and with what velocity will it strike an iron target 300 feet from the gun? *Ans.* I. V. = 2010 f. s.
S. V. = 1990 f. s.

3. The striking energy necessary for a 6-inch B. L. R. shot to just penetrate 12 inches of iron armor is 2559 foot-tons with a 100-pound projectile, 1 ton = 2240 pounds, $g = 32.2$ feet. The gun is 300 feet from the armor plate; what is the necessary muzzle velocity? Two screens are between the gun and plate, are 100 feet apart, and the first is 100 feet from the gun; what is the time of flight between screens? *Ans.* S. V. = 1921.3 f. s.
M. V. = 1949.1.
T = .0516.

4. A 6-inch 100-pound rifled projectile is fired through two screens which are 100 feet apart; first being 100 feet from the muzzle of the gun. The time observed by chronoscope between screens is $\frac{10}{185}$ of a second. (1) What is the muzzle velocity? (2) What is the velocity at the second screen? (3) What is the total resistance of the air in pounds at the second screen? (4) What is the total energy with which it strikes an iron target 100 yards from the muzzle of the gun?

Ans. I. V. $=1863$.
$V_1=1845$.
$V_2=1836$.
Striking energy $=2337$ ft. tons.

5. A 6-inch 100-pound rifled projectile is fired through two screens which are 100 feet apart, the first of which is 100 feet from the muzzle of the gun; the time taken by the projectile to travel between the screens is $\frac{1}{20}$ of a second. (1) Find the muzzle velocity; (2) find the velocity at the second screen; (3) find the total resistance of the air in pounds at the second screen; (4) the total energy with which it strikes a vertical iron plate 100 yards from the muzzle.

Ans. M. V. $= 2014.7$.
$v = 1995$ at second screen.
$q = 586$ pounds.
$E = 2732$ ft. tons.

6. A 6-pdr. R. F. gun is mounted 24.5′ above the water, and the axis of the bore is set by quadrant at an angle of 4° with the horizontal. When fired with an initial velocity of 1804 f. s., jump + 2′ 10″, barometer 30.29, thermometer 81° F., the range on the water was observed to be 2780.4 yards. For this gun $d = 2.24''$, $w = 6$ lbs., and $i = 0.9$. It is required to find the correction to be applied to the observed range for variations from the standard conditions of barometer and thermometer. What is the angle between the line of sight and the line of departure in this particular case?

Ans. Cor. — 19.1 yds.
4° 12′ 16″.

7. (*a*) A 12″ projectile, weight 850 lbs., of standard form is fired through two screens 100 feet apart, the first screen being 100 feet from the gun. The time of flight between screens is observed by chronograph to be 0.047 seconds; what is the muzzle velocity?

(*b*) The jump of this gun being 0°, how much above a certain mark on a steel plate 300 feet from the gun should the axis of the bore be pointed in order to hit the mark on the plate?

Ans. I. V. 2135 f. s.
3.84″.

8. The 5″ B. L. R. was fired at the Proving Ground, Dec. 7, 1887, when the barometer was 30.38 and the thermometer 48° F. The muzzle of the gun was 5.375′ above the water and the axis of the bore was set by quadrant at an angle of 5° 25′ with the horizontal. The jump was + 3′, I. V. 2000 f. s., $w = 60$ lbs., and the range on the water was observed to have been 4441 yards: It is required to find the correction to apply to the observed range for variations from the standard conditions of barometer and thermometer and also to find the *angle* between the line of sight and the line of departure in this particular case. *Ans.* Cor. + 47.3 yds.
5° 29′ 30″.

9. The velocity of a flat-headed rifle projectile, $d = 6''$, $w = 70$ lbs., when fired through two screens 150 feet apart was found to be 1881 f. s. and 1835 f. s. at the screens, standard conditions of the air. Find the value of i, the coefficient of form in this case.

Ans. $i = 2.438$.

CHAPTER V.

TRAJECTORIES IN AIR.*

§ I.

From the hypothesis that the resistance is directly opposed to the velocity, it follows immediately that the trajectory is plane. In fact, if a vertical plane be passed through any element of it, that plane will contain the two forces which act upon the projectile, namely, gravity and the resistance of the air, their resultant will lie in the same plane, and consequently also the consecutive element. Hence all the elements lie in the same vertical plane.

Differential Equations.

Let the centre of gravity of the projectile be referred to two axes, x, y, lying in the plane of fire, the first horizontal, directed toward the motion; the second vertical, in the direction opposed to gravity.

Let

w denote the weight of the projectile,
g " the acceleration of gravity,
v " the velocity,
φ " the angle made by its direction with the axis of x,
$f(v)$ " the retardation,
t " the time:

we shall have then

$$\frac{w}{g}\frac{d(v\cos\varphi)}{dt} = -\frac{w}{g}f(v)\cos\varphi,$$

$$\frac{w}{g}\frac{d(v\sin\varphi)}{dt} = -\frac{w}{g}f(v)\sin\varphi - w,$$

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*Translated and adapted from "Balistica, di F. Siacci, Seconda Edizione," Turin, 1888, by Professor William Woolsey Johnson, U. S. Naval Academy, with the co-operation of Lieut.-Commander Sperry, Head of the Department of Ordnance and Gunnery, 1888. English units have been substituted for metric units, Mayevski's law of resistance has been used in place of that employed by Major Siacci.

or

$$\frac{d(v\cos\varphi)}{dt}=-f(v)\cos\varphi, \tag{1}$$

$$\frac{d(v\sin\varphi)}{dt}=-f(v)\sin\varphi-g. \tag{2}$$

Multiplying (1) by $v\sin\varphi$, (2) by $v\cos\varphi$, and subtracting the first result from the second, we have

$$\frac{v\cos\varphi\,d(v\sin\varphi)-v\sin\varphi\,d(v\cos\varphi)}{dt}=-gv\cos\varphi;$$

furthermore, multiplying both members of this equation by $\frac{dt}{v^2\cos^2\varphi}$, we obtain

$$d\frac{v\sin\varphi}{v\cos\varphi}=-\frac{g}{v}\frac{dt}{\cos\varphi},$$

and therefore (since $d\tan\varphi=\sec^2\varphi d\varphi$),

$$gdt=-v\frac{d\varphi}{\cos\varphi}. \tag{3}$$

Finally, substituting this value of dt in (1) and in

$$\frac{dx}{dt}=v\cos\varphi,\quad \frac{dy}{dt}=v\sin\varphi,\quad \frac{ds}{dt}=v,$$

we derive

$$gd(v\cos\varphi)=f(v)\,vd\varphi, \tag{4}$$

$$\left.\begin{aligned} gdx&=-v^2d\varphi,\\ gdy&=-v^2\tan\varphi d\varphi,\\ gds&=-v^2\sec\varphi d\varphi.\end{aligned}\right\} \tag{5}$$

If the integration of (4) were possible, that is, if it were possible to derive a finite relation between v and φ, one of these variables might be eliminated from equations (3) and (5), and then it would become possible to obtain t, x, y and s, either exactly or by means of quadrature.

But the form which we can give to $f(v)$ compatibly with the results of experiment upon the resistance do not permit such integration. It is consequently necessary to have recourse to methods of approximation. The following method gives results which are the more exact the more direct the fire, that is to say, the smaller the angle of departure; it moreover gives a sufficient approximation even in cases of curved fire.

Ballistic Formulas.

Substituting in the differential equations the value of $f(v)$,* we have

$$gd(v\cos\varphi)=\frac{\delta i}{C}F(v)\,vd\varphi,$$

$$gdx=-v^2d\varphi, \qquad gdt=-v\sec\varphi d\varphi.$$

Let us decompose the velocity into two components, one vertical and the other parallel to the line of departure. Denoting this second component by z, we shall have (θ being the angle of departure)

$$v\cos\varphi=z\cos\theta. \tag{1}$$

The quantity z will be called the *pseudo-velocity.* Its value coincides with that of v at two points; at the origin and at the point in the descending branch where $\varphi=-\theta$. In the rest of the trajectory the pseudo-velocity differs so much the less from the velocity as the fire is more direct.

Substituting, we obtain

$$gdz=\frac{\delta iF(v)}{C}z\frac{d\varphi}{\cos\varphi},$$

$$gdx=-z^2\cos^2\theta\frac{d\varphi}{\cos^2\varphi},$$

$$gdt=-z\cos\theta\frac{d\varphi}{\cos^2\varphi}.$$

Let us now put

$$F(v)=\beta F(z)\frac{\cos^2\theta}{\cos\varphi}, \tag{2}$$

and thus derive from the first equation

$$gdz=\frac{\delta i\beta\cos^2\theta}{C}zF(z)\frac{d\varphi}{\cos^2\varphi},$$

whence

$$\frac{d\varphi}{\cos^2\varphi}=\frac{C}{\delta i\beta\cos^2\theta}\frac{gdz}{zF(z)}; \tag{3}$$

---

*The retardation of a projectile is inversely proportional to its ballistic coefficient: we therefore put

$$f(v)=\frac{\delta i}{C}F(v),$$

so that $F(v)$ is a function independent of the form, weight and dimensions of the projectile, and of the density of the air. In Chapter IV this function was denoted by $Af(v)$.

and, putting this value in the other two equations, there results

$$dx = -\frac{C}{\delta i\beta}\frac{z\,dz}{F(z)}, \tag{4}$$

$$dt = -\frac{C}{\delta i\beta\cos\theta}\frac{dz}{F(z)}. \tag{5}$$

The quantity β defined by (2), that is,

$$\beta = \frac{F(v)}{F(z)}\frac{\cos\varphi}{\cos^2\theta},$$

is a variable quantity, which varies however between limits which are the more restricted the more direct the fire. Thus, if the trajectory were rectilinear and horizontal, we should have $\theta = \varphi = 0$, $v = z$ and $\beta = 1$. Thus, assuming β to be a constant quantity, the flatter the trajectory the smaller will be the error. We shall speak further on of the most convenient value to be given to this quantity; in the meantime we shall put

$$\frac{C}{\delta i\beta} = C',$$

and we shall call C' *the reduced ballistic coefficient.*

Integrating equations (3), (4) and (5), putting

$$-\int\frac{z\,dz}{F(z)} = S(z), \quad -\int\frac{2g\,dz}{zF(z)} = I(z), \quad -\int\frac{dz}{F(z)} = T(z), \tag{6}$$

(or, as we shall generally write them, S_z, I_z, T_z*) and noticing that at the origin $z = V$, we shall have

$$\tan\varphi - \tan\theta = -\frac{C'}{2\cos^2\theta}[I(z) - I(V)], \tag{7}$$

$$x = C'[S(z) - S(V)], \tag{8}$$

$$t = \frac{C'}{\cos\theta}[T(z) - T(V)]. \tag{9}$$

Moreover, multiplying (7) by the differential of (8), and observing that $dx\tan\varphi = dy$, we have

$$dy - dx\tan\theta = -\frac{C'^2}{2\cos^2\theta}[I(z) - I(V)]\,dS(z);$$

---

*It will be noticed that the functions S and T are the same as those which are otherwise obtained in Chapter IV and are tabulated under the headings S_V and T_V in Table I.

integrating, and putting

$$\int I(z)dS(z)\,,\quad\text{or}\quad -\int I(z)\frac{zdz}{F(z)}=A(z)\,,$$

there results

$$y-x\tan\theta=-\frac{C'^2}{2\cos^2\theta}[A(z)-A(V)-I(V)\{S(z)-S(V)\}].$$

Finally, dividing by (8), we obtain

$$\frac{y}{x}-\tan\theta=-\frac{C'}{2\cos^2\theta}\left[\frac{A(z)-A(V)}{S(z)-S(V)}-I(V)\right]. \qquad (10)$$

Equations (7), (8), (9) and (10), together with (1), are the *ballistic formulas.*

The ballistic formulas may be grouped together as follows:

$$x=C'[S_z-S_V]\,,$$

$$\frac{y}{x}=\tan\theta-\frac{C'}{2\cos^2\theta}\left[\frac{A_z-A_V}{S_z-S_V}-I_V\right].$$

$$\tan\varphi=\tan\theta-\frac{C'}{2\cos^2\theta}[I_z-I_V]\,,$$

$$t=\frac{C'}{\cos\theta}[T_z-T_V]\,,$$

$$v=z\frac{\cos\theta}{\cos\varphi}.$$

They express the two coordinates, the inclination, the time and the velocity at any point of the trajectory directly as functions of a new variable, the pseudo-velocity; the reduced ballistic coefficient, the angle of departure and the initial velocity being supposed known.

The values of the I_z and the A_z functions, when integrated over small limits, have been computed and tabulated for the several laws of the resistance of the air, and for values of the velocity from 3600 f. s. to 500 f. s. These values will be found in Table I, in the columns headed I_v and A_v respectively.

The formulas used in the computation of the tabular values are as follows:

I_v.

$3600 \text{ ft.} > v > 1375 \text{ ft.}$:

$$I_v = [5.3880723]\frac{1}{v^2} - .01885677.$$

$1375 \text{ ft.} > v > 1230 \text{ ft.}$:

$$I_v = [8.3503422]\frac{1}{v^3} + .024218.$$

$1230 \text{ ft.} > v > 970 \text{ ft.}$:

$$I_v = [14.3075324]\frac{1}{v^5} + .0725069.$$

$970 \text{ ft.} > v > 790 \text{ ft.}$:

$$I_v = [8.5578016]\frac{1}{v^3} - .086888.$$

$790 \text{ ft.} > v > 500 \text{ ft.}$:

$$I_v = [5.83744405]\frac{1}{v^2} - .45621.$$

A_v.

$3600 \text{ ft.} > v > 1375 \text{ ft.}$:

$$A_v = [8.9677986]\frac{1}{v^2} + [2.5184393] \log v - 1245.2.$$

$1375 \text{ ft.} > v > 1230 \text{ ft.}$:

$$A_v = [14.7673997]\frac{1}{v^4} + [5.4032558]\frac{1}{v} - 66.246.$$

$1230 \text{ ft.} > v > 970 \text{ ft.}$:

$$A_v = [26.6025988]\frac{1}{v^8} + [11.5814144]\frac{1}{v^3} + 113.817.$$

$970 \text{ ft.} > v > 790 \text{ ft.}$:

$$A_v = [15.1823184]\frac{1}{v^4} - [6.1655366]\frac{1}{v} + 833.186.$$

$790 \text{ ft.} > v > 500 \text{ ft.}$:

$$A_v = [9.8670749]\frac{1}{v^2} + [4.3515049] \log v - 74006.48.$$

§ II.

The value of β.

As for the quantity β, it is convenient in the first place to distinguish the case in which the resistance increases more rapidly than the square of the velocity, from that in which it is proportional simply to the square of the velocity. Let us put $F(v) = v^2 K(v)$, so that in the first case $K(v)$ is an increasing function of v, and in the second case it is a constant. Then β becomes

$$\beta = \frac{v^2}{z^2} \frac{K(v)}{K(z)} \frac{\cos\varphi}{\cos^2\theta} = \frac{K(v)}{K(z)} \sec\varphi.$$

Now at the origin, $\varphi = \theta$, $z = v$, whence $\beta = \sec\theta > 1$. At the vertex, $\varphi = 0$, $z = v\sec\theta > v$, and therefore $\beta = \frac{K(v)}{K(z)}$, which is $<$ 1 in the first case. At the point in which the inclination φ becomes $= -\theta$, we have also $z = v$, and therefore $\beta = \sec\theta > 1$, and it remains > 1 for all the rest of the trajectory. We conclude therefore that β varies between two limits, of which one is > 1 and the other is < 1, when the resistance increases more rapidly than the square of the velocity. In the second case, we have $K(v) = \text{constant}$; therefore, the inferior limit of β being $= 1$, the mean value will in this case be > 1. It is important, however, to notice that the mean value* of β, which ought to be put in the formula to obtain results absolutely exact, is not always the same; it ought to vary not only with the data of the trajectory, but also with the problem itself.† Thus, given X and V, if we seek θ, the value which ought to be

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* If in a definite integral a variable is replaced by a constant, in order that the value of the integral may be unchanged, the value of the constant must be a ***mean value*** of the variable falling between the greatest and least values which occur within the limits of integration. See Int. Calc., Art. 97.

† That is to say, with the formula employed. The mean value defined as in the preceding foot-note is the mean value of the variable in question when considered as a function of the variable whose differential remains under the integral sign. For instance, equation (4), p. 4, may be written

$$\frac{\delta i}{C}\,dx = -\frac{1}{\beta}\frac{z\,dz}{F(z)} = \frac{1}{\beta}\,dS_z, \text{ whence } \frac{\delta i}{C}\,x = \int_{S_V}^{S_z} \frac{1}{\beta}\,dS.$$

In this case we are therefore concerned with the mean value of $\frac{1}{\beta}$ considered as a function of S. This is the same thing as the mean value when all values of S between S_V and S_z are considered equally probable. Compare Johnson's Least Squares, Art. 24.

taken for β is different from that which would occur if we sought ω. Therefore, wishing to adopt, as we intend to do, a single value for all problems regarding the same trajectory, it will be convenient to adhere to that which is adapted to the principal problem of fire, which is evidently this: *given two of the quantities X, V and θ, to determine the third.*

Now, if the resistance increased as the cube of the velocity, the ballistic formulas could be integrated exactly, and the exact formulas applied to the principal practical problem give results agreeing with those which are obtained, the resistance being supposed equal, from the approximate formulas, when a suitable value is put in place of β. This value, for $\theta = 20°$ and for cases of ordinary velocity, differs very little from unity; in fact, it differs only by .005 in cases of very great velocities, which are rarely combined in practice with the angle of 20°, much less with greater angles. For $\theta = 5°$, 10°, 15°, the value of β, according as the initial velocities are the smallest or the greatest which occur in practice, varies respectively from 0.999 to 1, from 0.998 to 1.001, from 0.996 to 1.004.*

Now the resistance of the air, although it follows a law very different from the cubic, may be regarded as varying *in the mean* as the cube of the velocity when the velocities are such as are in practice combined with angles not greater than 20°. Therefore we assume that *when the angle of departure is not greater than 20° we may put* $\beta = 1$.

When the resistance increases as the square of the velocity the differential equations may again be integrated exactly, and the formulas applied to the most important practical problem give results agreeing with those which, the resistances being supposed equal, are obtained from the approximate formulas, when for β is put a value which, for angles of departure less than 45°, differs very little from $(\sec\theta)^{\frac{1}{2}}$.

When the initial velocity is small the law of resistance approaches more nearly to the quadratic law, because the smaller the initial velocity the larger relatively is the portion of the trajectory in which this law of resistance prevails. With moderate ranges, low initial velocities are used in connection with considerable angles of departure; so that in practice the value of β should somewhat exceed unity in these cases; that is to say, when the angle of departure is near the limit, 20°, for which we have put $\beta = 1$.

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* Parodi: Sull' approssimazione delle formole balistiche. [Rivista d'Artiglierio e Genio, June 1887.]

Diminution of the Density of the Air.

In the integrations performed we have taken as constant the quantity δ, although the density of the air diminishes as the projectile rises. We ought therefore to put for δ in the formulas a value somewhat smaller than that corresponding to the horizon of the piece, which is directly determined from the indications of the barometer and thermometer. For small angles of departure this diminution of δ may be neglected. It becomes sensible when the angle of departure is in the neighborhood of 15° or 20°. In this case, however, we have seen in the preceding paragraph that, in adopting, as we have, unity for the value of β, we have taken a value which is somewhat too small. If then we take for δ the density of the air at the horizon of the piece, which is too great, we shall make a compensation for the value of β which is too small, since δ and β are multiplied together in the expression $C' = \frac{C}{\delta i \beta}$.

§ III.

Problems of Fire.

In the formulas for the solution of problems by means of Ingersoll's Ballistic Tables, $C' = \frac{C}{\delta i \beta} = \frac{w}{d^2 \delta i \beta}$, in which w is the weight of the projectile in pounds and d the diameter of the bore in inches. In the problems discussed below we assume $\beta = 1$, the angles of departure being all supposed less than 20°. The value of i is unity for the standard navy shells: it depends principally upon the form of the head, and for certain forms may be taken as in the following table, the ratios of the numbers being the same as those given on page 26:

VALUES OF i.

For ogival heads, radius 2 calibres,	1.
" " " " 1 calibre,	1.0527
" hemispherical heads,	1.2755
" flat "	1.9288

The method of determining i experimentally for any form of projectile, and that of deriving δ from the readings of the barometer and thermometer, have been given in Chapter IV.

In the illustrative examples we have assumed δ, i and β each equal to unity, so that $C' = C = \frac{w}{d^2}$.

Before proceeding to the cases which usually arise in practice, it may be useful to give an illustration of the direct use of the formulas in determining the character of a given trajectory at several widely separated points. For this purpose let us take the trajectory described by a 100-pound shell fired from a 6-inch B. L. R. with the initial velocity of 2000 f. s. and with an angle of departure of 4°; and let us examine its character at the horizontal distances of 1000 yds., 3000 yds. and 4000 yds. from the origin.

The formulas for use are nearly as written on page 51; namely:

$$S_z = S_V + \frac{x}{C}, \tag{1}$$

$$y = x \tan\theta - \frac{Cx}{2\cos^2\theta}\left[\frac{A_z - A_V}{S_z - S_V} - I_V\right], \tag{2}$$

$$\tan\varphi = \tan\theta - \frac{C}{2\cos^2\theta}\left[I_z - I_V\right], \tag{3}$$

$$t = \frac{C}{\cos\theta}\left[T_z - T_V\right], \tag{4}$$

$$v = z\frac{\cos\theta}{\cos\varphi}, \tag{5}$$

by which, V, θ, C and x being given, the corresponding values of z, y, φ, t and v are successively found.

In the example $V = 2000$, $\theta = 4°$, $C = \frac{100}{36}$, and for $x = 1000$ yds. $= 3000$ ft. we have $\frac{x}{C} = 1080$. From the table we take

$$S_V = 4466.2 \qquad A_V = 76.26 \qquad I_V = 0.04224 \qquad T_V = 1.689.$$

Therefore, by equation (1), $S_z = 5546.2$; whence, by the table,

$$z = 1735.0 \qquad A_z = 132.21 \qquad I_z = 0.06232 \qquad T_z = 2.269$$

and

$$\Delta A = 55.95 \qquad \Delta I = 0.02008 \qquad \Delta T = 0.580,$$

where, for shortness, we have writen ΔA for $A_z - A_V$, etc.

We proceed to calculate y by equation (2),

$$\begin{array}{rl} \log \Delta A = & 1.74780 \\ \log \dfrac{x}{C} = \log \Delta S = & 3.03342 \\ \log \dfrac{\Delta A}{\Delta S} = & 8.71458 - 10 \qquad \dfrac{\Delta A}{\Delta S} = 0.051830 \\ & \qquad\qquad\qquad\qquad I_V = 0.04224 \\ \log \left[\dfrac{\Delta A}{\Delta S} - I_V\right] = & 7.98182 - 10 \qquad \dfrac{\Delta A}{\Delta S} - I_V = \ .00959 \\ \log \tfrac{1}{2} C = & 0.14267 \\ \log x = & 3.47712 \\ 2 \log \sec \theta = & 0.00212 \\ & 1.60373 \ . \quad . \quad . \qquad 40.154 \\ \log x = & 3.47712 \\ \log \tan \theta = & 8.84464 - 10 \\ \log x \tan \theta = & 2.32176 \ . \quad . \quad . \qquad 209.776 \\ & \qquad\qquad\qquad\qquad y = 169.622 \end{array}$$

To find φ we have, employing equation (3),

$$\begin{array}{rl} \log \Delta I = & 8.30276 - 10 \\ \log \tfrac{1}{2} C = & 0.14267 \\ 2 \log \sec \theta = & 0.00212 \\ & 8.44755 - 10 \ . \quad . \quad .0280025 \\ \log \tan \theta = & 8.84464 - 10 \qquad \tan \theta = .069927 \\ \log \tan \varphi = & 8.62223 - 10 \qquad \tan \varphi = .041902 \\ & \qquad\qquad\qquad\qquad \varphi = 2^\circ \ 24'. \end{array}$$

To find t from equation (4),

$$\begin{array}{rl} \log \Delta T = & 9.76343 - 10 \\ \log C = & 0.44370 \\ \log \sec \theta = & 0.00106 \\ \log t = & 0.20819 \qquad\qquad t = 1.6151. \end{array}$$

Finally, from equation (5), we have for v

$$\begin{array}{rl} \log z = & 3.23930 \\ \log \cos \theta = & 9.99894 - 10 \\ \log \sec \varphi = & 0.00038 \\ \log v = & 3.23862 \qquad\qquad v = 1732.3. \end{array}$$

The original horizontal velocity is $V \cos \theta = 1995.12$. The loss of horizontal velocity in the time t, which is due entirely to the resistance of the air, is about 267 feet. The time of describing the horizontal distance, which would have been 1.5037 seconds in vacuo, has been increased by the resistance to 1.6151 seconds.

The actual amount of fall from the tangent at the origin or line of departure, as found in the computation for y, is 40.154 feet, while the fall on an unresisted body in the time t, or $\frac{1}{2}gt^2$, is about 41 feet.* On the other hand, at the same horizontal distance, the parabola which would be described if the resistance of the air did not exist, is below the tangent 36 feet, which is the value of $\frac{1}{2}gt^2$ when $t = 1.5037$. Thus the trajectory is, at this point, 4 feet below the parabola.

Taking in the same trajectory $x = 3000$ yds. $= 9000$ ft., we have $S_z = 7706.2$, whence $z = 1307.4$,

$$A_z = 327.68 \qquad I_z = 0.12447 \qquad T_z = 3.709.$$

Proceeding in the same manner we shall find

$$y = 185.56, \quad \varphi = -2^\circ\, 34', \quad t = 5.6249, \quad v = 1305.5,$$

the distance below the line of departure being 443.78 ft., which is about 63 feet less than the free fall in the time t. The amount of retardation in time is now 1.1 seconds, and the distance below the parabola will be found to be 118 feet. The negative value of φ shows that the projectile is now in the descending branch.

Finally, if we take $x = 4000$ yds. $= 12{,}000$ ft., we shall have $S_z = 8786.2$, whence $z = 1156.2$, and we shall find

$$y = -42.00, \quad \varphi = -6^\circ\, 15', \quad t = 8.0726, \quad v = 1160.3.$$

The projectile is now below the horizon of the piece, although the corresponding point of the parabola is still 160 feet above the same plane. The retardation in time is now over two seconds.

Practical Problems.

CASE I.

Given the initial velocity and the range, to find the angle of departure, the angle of fall, the time of flight, and the final velocity.

Let X denote the range, ω the angle of fall, T the time of flight, and f. v. the final velocity. These are the values of $x, -\varphi$, t and v respectively, corresponding to $y=0$. Let Z denote the corresponding pseudo-velocity. Putting $y=0$ in the second of the ballistic formulas, and $\varphi=-\omega$ in the third, we have

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* Of course the projectile is at an altitude much less than that it would have reached had it been unresisted, but it is not so far below the line of departure, because, except at the origin, the resistance acts in a direction whose inclination is less than that of the line of departure.

$$\tan\theta=\frac{C}{2\cos^2\theta}\left[\frac{A_Z-A_V}{S_Z-S_V}-I_V\right],$$

$$\tan\omega=\frac{C}{2\cos^2\theta}\left[I_Z-I_V\right]-\tan\theta.$$

Hence we have, for this case, the following formulas, of which the second and third are derived from the two equations just written:

$$S_Z=S_V+\frac{X}{C}, \tag{1}$$

$$\sin 2\theta=C\left[\frac{A_Z-A_V}{S_Z-S_V}-I_V\right], \tag{2}$$

$$\tan\omega=\frac{C}{2\cos^2\theta}\left[I_Z-\frac{A_Z-A_V}{S_Z-S_V}\right], \tag{3}$$

$$T=\frac{C}{\cos\theta}\left[T_Z-T_V\right], \tag{4}$$

$$\text{f. v.}=\frac{Z\cos\theta}{\cos\omega}. \tag{5}$$

When θ, as found by (2), is small (say less than 5°), the difference between the values of $\cos\omega$ and $\cos\theta$ is so small relatively to that of $\cos\theta$ that the value of $\frac{\cos\omega}{\cos\theta}$ is very close to unity. Thus, multiplying equation (3) by $\frac{\cos^2\omega}{\cos^2\theta}=1$, we have the approximate formula

$$\sin 2\omega=C\left[I_Z-\frac{A_Z-A_V}{S_Z-S_V}\right].$$

Example: A shell weighing 500 pounds is fired from a 10-inch B. L. R. with an initial velocity of 2000 f. s. What must be the angle of elevation to obtain a range of 10,000 yds., the jump being $+5'$? Calculate also the angle of fall, time of flight, and final velocity.

We have in this example $C=5$, $X=30{,}000$, $\frac{X}{C}=6000$,

$S_V=4466.2$ $A_V=76.26$ $I_V=0.04224$ $T_V=1.689$.

Thus, by equation (1), $S_Z=10466.2$; whence by Table I,

$$\begin{array}{llll} Z=1011.5 & A_Z=847.84 & I_Z=0.26424 & T_Z=6.151 \\ \text{and} & \Delta A=771.58 & \Delta I=0.22200 & \Delta T=4.462 \end{array}$$

To calculate θ we have

$$\begin{array}{rl} \dfrac{\Delta A}{\Delta S} & =0.12860 \\ I_V & =\underline{0.04224} \\ & 0.08636 \\ C & =\underline{5} \\ \sin 2\theta & =0.43180 \qquad\qquad 2\theta=25^\circ\,34'\,56'' \end{array}$$

$$\theta=12^\circ\,47'\,28''.$$

Subtracting the jump, we have for the angle of elevation

$$\alpha=12^\circ\,42\tfrac{1}{2}'.$$

For ω we have

$$\begin{array}{rl} I_Z & =0.26424 \\ \dfrac{\Delta A}{\Delta S} & =\underline{0.12860} \\ I_Z-\dfrac{\Delta A}{\Delta S} & =0.13564 \quad . \quad . \quad . \quad 9.13239-10 \\ & \qquad\qquad \log\tfrac{1}{2}C=0.39794 \\ & \qquad\qquad 2\log\sec\theta=\underline{0.02182} \\ & \qquad\qquad \log\tan\omega=9.55215-10 \quad \omega=19^\circ 37\tfrac{1}{2}'. \end{array}$$

To find T by equation (4), since $\Delta T=4.462$

$$\begin{array}{ll} C\Delta T=22.31 & \log C\Delta T=1.34850 \\ & \log\sec\theta=\underline{0.01091} \\ & \log T=1.35941 \qquad T=22.877. \end{array}$$

Lastly, for the final velocity, we have

$$\begin{array}{rl} \log Z & =3.00496 \\ \log\cos\theta & =9.98909-10 \\ \log\sec\omega & =\underline{0.02599} \\ \log \text{f. v.} & =3.02004 \qquad\qquad \text{f. v.}=1047.2. \end{array}$$

Example 2.—Given d, 10″; w, 500 lbs.; initial velocity, 2000 f. s.; jump, + 5′; range, 2533 yds.; calculate the angles of elevation and fall and the time of flight.

Ans. α, 1° 55′ 44″; ω, 2° 17′ 41″; T, 4.2072 sec.

Example 3.—Given d, 8″; w, 250 lbs.; initial velocity, 1700 f. s.; range, 3500 yds.; jump. 0; calculate α, ω, T, and the final velocity.

Ans. α, 4° 19′; ω, 5° 27′; T, 7.4 sec.; f. v. 1207 f. s.

Example 4.—Given d, 5″; w, 60 lbs.; initial velocity, 2000 f. s.; range, 3910 yds.; jump, + 2′ 47″; calculate θ, α, ω, T, and the final velocity.

Ans. θ, 4° 21′ 5″; α, 4° 18′ 18″; ω, 6° 29′ 26″; T, 8.1916ˢ; f.v. 1102.5 f.s.

Example 5.—Given d, 6″; w, 100 lbs.; initial velocity, 2000 f. s.; range, 4000 yds.; jump, — 7′ 00″; calculate the angle of elevation.

Ans. 4° 19′ 10″.

Example 6.—Given d, 6″; w, 100 lbs.; initial velocity, 2000 f. s.; range, 3200 yds.; jump, — 7′ 00″; calculate the angle of elevation.

Ans. 3° 11′ 22″.

Example 7.—Given d, 6″; w, 100 lbs.; initial velocity, 2000 f. s.; range, 5000 yds.; jump, — 7′ 00″; calculate the angle of elevation.

Ans. 5° 59′ 35″.

Example 8.—Given d, 6″; w, 100 lbs.; initial velocity, 2000 f. s.; range, 7000 yds.; jump, — 7′ 00″; calculate the angle of elevation.

Ans. 10° 17′ 5″.

CASE II.

Given the initial velocity and the angle of departure, to find the coordinates of the vertex and the corresponding time and velocity.

Let z_0 denote the value of z at the vertex, at which point $\varphi = 0$, and let x_0, Y_0, t_0, and v_0 be the corresponding values of x, y, t and v so that Y is the greatest height reached by the projectile. Putting $\varphi = 0$ in the third of the ballistic equations, p. 51, we have

$$\tan\theta = \frac{C}{2\cos^2\theta}\left[I_{z_0} - I_V\right],$$

from which z_0 is to be found; hence the formulas for use are

$$I_{z_0} = I_V + \frac{\sin 2\theta}{C}, \qquad (1)$$

$$x_0 = C\left[S_{z_0} - S_V\right], \qquad (2)$$

$$Y = x_0 \tan\theta - \frac{Cx_0}{2\cos^2\theta}\left[\frac{A_{z_0} - A_V}{S_{z_0} - S_V} - I_z\right], \qquad (3)$$

$$t_0 = \frac{C}{\cos\theta}\left[T_{z_0} - T_V\right], \qquad (4)$$

$$v_0 = z_0 \cos\theta. \qquad (5)$$

Example: Find the coordinates of the vertex, etc., in the example on page 59 (θ having been found); that is to say, given

$$V=2000 \qquad C=5 \qquad \theta=12^\circ\ 47\tfrac{1}{2}'.$$

By equation (1) we have

$$\begin{array}{lll} 2\theta=25^\circ\ 35' & \sin 2\theta=0.43182 & \dfrac{\sin 2\theta}{C}=0.086364 \\ & & I_V=\underline{0.04224} \\ & & I_{z_0}=0.12860; \end{array}$$

whence, by the table,

$$\begin{array}{llll} z_0=1289.9 & S_{z_0}=7814.8 & A_{z_0}=341.38 & T_{z_0}=3.7925 \\ & S_V=\underline{4466.2} & A_V=\underline{\ \ 76.26} & T_V=\underline{1.689} \\ & \varDelta S=3348.6 & \varDelta A=265.12 & \varDelta T=2.103. \end{array}$$

By equation (2), $x_0=C\varDelta S$ $\therefore$ $x_0=16743$ ft. $=5581$ yds.

To compute Y by equation (3),

$$\begin{array}{rl} \log \varDelta A= & 2.42343 \\ \log \varDelta S= & \underline{3.52487} \\ & 8.89856-10 \quad \dots \quad \dfrac{\varDelta A}{\varDelta S}=0.079170 \\ & \qquad\qquad\qquad\qquad I_V=\underline{0.042244} \\ & 8.56733-10 \quad \dots \quad \dfrac{\varDelta A}{\varDelta S}-I_V=0.036926 \\ \log \tfrac{1}{2}C= & 0.39794 \\ \log x_0= & 4.22384 \\ 2\log\sec\theta= & \underline{0.02183} \\ & 3.21094 \quad \dots\dots \quad 1625.3 \\ \log x_0= & 4.22384 \\ \log\tan\theta= & \underline{9.35610}-10 \\ & 3.57994 \quad \dots \quad x_0\tan\theta=\underline{3801.4} \\ & \qquad\qquad\qquad\qquad Y=2176.1. \end{array}$$

To find t_0 we have

$$\begin{array}{rl} \log \varDelta T= & 0.32284 \\ \log C= & 0.69897 \\ \log\sec\theta= & \underline{0.01091} \\ \log t_0= & 1.03272 \qquad\qquad t_0=10.782. \end{array}$$

Finally, for v_0,

$$\begin{aligned} \log z_0 &= 3.11056 \\ \log \cos \theta &= 9.98909 - 10 \\ \hline \log v_0 &= 3.09965 \qquad\qquad v_0 = 1257.9. \end{aligned}$$

These results, and those found for the same data on p. 60, enable us to compare the loss of velocity experienced, the horizontal distance covered, and the time consumed in describing the ascending and the descending branch of this trajectory. They are as follows, the initial velocity being 2000 f. s.:

	Loss of Velocity.	Horizontal Distance.	Time Elapsed.
Ascending branch,	742 f. s.	5581 yds.	10.78 sec.
Descending "	211 f. s.	4419 yds.	12.10 sec.

The considerable difference between the horizontal projections of the two branches and the small difference between the times should be noticed. The great difference in the rate of retardation in the two branches is also noteworthy, and a comparison of the corresponding losses of energy is still more striking. The kinetic energy of the shell at the origin, the vertex and the point of fall are respectively about 31 million foot-pounds, 12½ millions, and 8½ millions. There is also at the vertex, however, about one million foot-pounds of potential energy.

Example 2.—Given d, 6″; w, 100 lbs.; initial velocity, 2000 f. s.; θ, 4°; find the coordinates of the vertex and the time of flight to that point. *Ans.* x_0, 2106.6 yds.; Y, 81.32 yds.; t_0, 3.692 sec.

Example 3.—Given d, 10″; w, 500 lbs.; initial velocity, 2000 f. s.; θ, 2°; find the coordinates of the vertex and the time of flight to that point. *Ans.* x_0, 1308.2 yds.; Y, 23.7 yds.; t_0, 2.056 sec.

Example 4.—Given d, 10″; w, 500 lbs.; initial velocity, 2000 f. s.; θ, 20°; find the coordinates of the vertex.

Ans. x_0, 7201.3 yds.; Y, 1548.1 yds.

Example 5.—Given d, 10″; w, 500 lbs.; initial velocity, 2000 f. s.; θ, 20°; find the abscissa of the point at which $\varphi = 5°$ and the velocity of the shell at this point. *Ans.* x, 6051.3 yds.; v, 1174.9 f. s.

Example 6.—The velocity of a 10-inch shell weighing 500 pounds at the vertex is 1804 f. s., and the angle of fall is 2° 17′; find the ordinate of the vertex and the distance from its foot to the point of fall. *Ans.* Y, 23.7 yds.; $X-x_0$, 1225.1 yds.

CASE III.

Given the initial velocity and the angle of departure, to find the range, the time of flight, angle of fall and final velocity.

It is necessary in this case to find Z, the pseudo-velocity at the point of fall, from the equation $y=0$, which, as on p. 51, gives

$$\sin 2\theta = C\left[\frac{A_Z-A_V}{S_Z-S_V}-I_V\right].$$

This equation contains two functions of the unknown quantity, Z; hence we cannot find it directly from the ballistic table, but must have recourse to the method of trial and error.

Writing the equation in the form

$$\frac{A_Z-A_V}{S_Z-S_V}=I_V+\frac{\sin 2\theta}{C}, \tag{1}$$

in which the second member is a known quantity, we assume two values of Z, and calculate the corresponding values of the first member. Of course neither of these will be found to agree exactly with the known value of the second member; but, if the values of Z are sufficiently near to the true one, a value very nearly correct may be found by the assumption that small variations in the value of Z produce proportional variations in the value of the function

$$\frac{A_Z-A_V}{S_Z-S_V}.$$

This will be illustrated by the example below, but we have first to show how the two assumed values of Z are to be taken.

Approximate Value of Z.

An approximate value of Z may be obtained as follows: If we neglect the vertical component of the resistance of the air, which is small, especially with low angles of departure, the time t_0, of describing the ascending branch, and the time, $T-t_0$, of describing the descending branch, will each be equal to the time occupied by a body falling freely (in vacuo) from the height of the vertex, Y. Thus an approximate value of the time integral, T_V, may be found by putting $T = 2t_0$.

Substituting the values of T and t_0 as expressed by the fourth of the ballistic equations, p. 51, we have, for the approximate value of T_Z,

$$T_Z = 2\,T_{z_0} - T_V. \tag{2}$$

The vertical component of the resistance acts in the direction of gravity in the ascending branch, and in that opposed to gravity in the descending branch. Hence the time t_0 is in reality less, and the time $T - t_0$ is greater, than the time of free fall mentioned above. It follows that $T > 2t_0$; hence the value of T_Z in (2) is always too small.

If then we put

$$T - t_0 = \rho t_0,$$

we know that ρ exceeds unity. Substituting, we have

$$T_Z - T_{z_0} = \rho\,(T_{Z_0} - T_V),$$

or

$$T_Z = T_{z_0} + \rho\,(T_{z_0} - T_V). \tag{3}$$

It is found that the excess of ρ over unity is not far from $\frac{1}{10}$ for angles of departure near 5° and the usual initial velocities, and that it is somewhat greater for higher angles. For instance, in the example solved above, the comparison made on p. 63 shows that the excess is about $\frac{1}{8}$; hence a close approximation to the value of T_Z will be found by adding to the value in equation (2) about one tenth of the difference $T_{z_0} - T_V$.

We have therefore the following method of finding two trial values of Z to use in equation (1): Find the value of I_{z_0}, as in Case II, by the equation

$$I_{z_0} = I_V + \frac{\sin 2\theta}{C}, \tag{4}$$

thence take the value of T_{z_0} from the table. Add to this the value of $T_{z_0} - T_V$ for one value of T_Z, and add to the result $\frac{1}{10}$ of the same quantity for another.

It is of course useless to obtain the exact values of Z corresponding to these two values of T_Z, which are merely guides in selecting by inspection of the table two trial values of Z, recollecting that the first value of T_Z is certainly too small, and the second is not far from the true value.

Having selected the trial values of Z, we next substitute them in equation (1), in which it will be noticed that the second member is the value of I_{z_0} already found, so that we may write the equation

$$\frac{A_Z-A_V}{S_Z-S_V}=I_{z_0}. \tag{5}$$

When the adjusted value of Z which satisfies this equation is found, the solution is completed as in other cases.

Examples: A shell weighing 250 pounds is fired from an 8-inch B. L. R. with the initial velocity 1700 f. s. and the angle of departure 4° 55′. Calculate the range, time of flight, angle of fall and final velocity.

We have here $C=\frac{250}{64}=\frac{1000}{256}$, $\log C=0.59176$,

also $\theta=4^\circ\ 55'$.

and from $V=1700$,

$$S_V=5701.3 \qquad A_V=142.13 \qquad I_V=0.06570 \qquad T_V=2.359.$$

To calculate I_{z_0} by equation (4),

$$\begin{array}{rlr}
\log\sin 2\theta = & 9.23244-10 & \\
\log C = & 0.59176 & \\
\hline
 & 8.64068-10 \quad . \quad . & \dfrac{\sin 2\theta}{C}=0.04372 \\
 & & I_V=0.06570 \\
\cline{3-3}
 & & I_{z_0}=0.10942.
\end{array}$$

For the approximate value of T_Z we have

$$\begin{array}{rcrl}
T_{z_0}=3.39 & \qquad & T_{z_0}=3.39 & \\
T_V=2.36 & & T_{z_0}-T_V=1.03 & \\
\cline{1-1}\cline{3-3}
T_{z_0}-T_V=1.03 & & T_Z=4.42 & \text{1st value.} \\
 & & .10 & \\
\cline{3-3}
 & & 4.52 & \text{2d value.}
\end{array}$$

Let us therefore take for the trial values of Z, $Z_1=1178$ and $Z_2=1164$, which correspond nearly to these two values of T_Z.

For $Z = 1178$, we have in equation (5)

$$\begin{array}{lll}
S_z = 8600.6 & A_z = 455.14 & \log \Delta A = 2.49555 \\
S_v = \underline{5701.3} & A_v = \underline{142.13} & \log \Delta S = \underline{3.46228} \\
\Delta S = 2899.3 & \Delta A = 313.01 & \log \dfrac{\Delta A}{\Delta S} = 9.03327 - 10.
\end{array}$$

For $Z = 1164$, we have

$$\begin{array}{lll}
S_z = 8718.2 & A_z = 474.47 & \log \Delta A = 2.52158 \\
S_v = \underline{5701.3} & A_v = \underline{142.13} & \log \Delta S = \underline{3.47956} \\
\Delta S = 3016.9 & \Delta A = 332.34 & \log \dfrac{\Delta A}{\Delta S} = 9.04202 - 10.
\end{array}$$

Now $\qquad \log I_{x_0} = 9.03910 - 10;$

thus the first result is too small, showing that 1178 is too large, and the second is too large, showing that 1164 is too small. We may assume that differences in the value of Z are proportional to the differences they produce in the values of $\log \frac{\Delta A}{\Delta S}$. The difference between the values found is 875, and that between the last and the true value is 292: hence, the difference between the trial values being 14, the correction to $Z = 1164$ is

$$\frac{292}{875} \text{ of } 14 = 4.7.$$

Thus the corrected value is

$$Z = 1168.7.$$

Repeating the process with this value of Z, we have

$$\begin{array}{lll}
S_z = 8678.1 & A_z = 467.79 & \log \Delta A = 2.51276 \\
S_v = \underline{5701.3} & A_v = \underline{142.13} & \log \Delta S = \underline{3.47375} \\
\Delta S = 2976.8 & \Delta A = 325.66 & \log \dfrac{\Delta A}{\Delta S} = 9.03901 - 10.
\end{array}$$

The result is so close to the value of $\log I_{x_0}$ that, considering the uncertainty of the last figures in the computation, we are not justified in making a new correction; but, if we suspect that it will be necessary to do so, it is best to make the calculation for the nearest value of Z given in the table (in order to avoid interpolation), and then correct this last value in the manner illustrated above.

We now find X by the formula

$$X = C[S_Z - S_V].$$

$\log \Delta S = 3.47375$
$\log C = 0.59176$
$\log X = 4.06551$ $\qquad X = 11628$ ft. $= 3876$ yds.

We find ω, as in Case I, by the formula

$$\tan \omega = \frac{C}{2 \cos^2 \theta}\left[I_Z - \frac{A_Z - A_V}{S_Z - S_V}\right],$$

or, as we may now write it,

$$\tan \omega = \frac{C}{2 \cos^2 \theta}[I_Z - I_{z_0}].$$

$I_Z = 0.16562$
$I_{z_0} = 0.10942$
$I_Z - I_{z_0} = 0.05619$ $\qquad \log(I_Z - I_{z_0}) = 8.74966 - 10$
$\log C = 0.59176$
$\log \frac{1}{2} = 9.69897 - 10$
$\theta = 4° 55'$ $\qquad 2 \log \sec \theta = 0.00320$
$\log \tan \omega = 9.04359 - 10$ $\qquad \omega = 6° 18' 32''$.

T is found by the formula

$$T = \frac{C}{\cos \theta}[T_Z - T_V].$$

$T_Z = 4.496$
$T_V = 1.359$
$\Delta T = 2.137$ $\qquad \log \Delta T = 0.32980$
$\log C = 0.59176$
$\log \sec \theta = 0.00160$
$\log T = 0.92316$ $\qquad T = 8.378$.

The final velocity is found by the formula

$$\text{f. v.} = Z \frac{\cos \theta}{\cos \omega}.$$

$Z = 1168.7$ $\qquad \log Z = 3.06770$
$\log \cos \theta = 9.99840 - 10$
$\log \sec \omega = 0.00264$
$\log \text{f. v.} = 3.06874$ $\qquad$ f. v. $= 1171.5$.

The time in the descending branch exceeds that in the ascending branch, in this example, by about $\frac{1}{15}$.

Example. 2.—Given d, 6″; w, 100 lbs.; θ, 4°; initial velocity, 2000 f. s.; find the range and the angle of fall.

Ans. X, 3866.5 yds.; ω, 5° 42′ 26″.

Example 3.—Given d, 5″; w, 60 lbs.; θ, 3°; initial velocity, 2000 f. s.; find the range and the angle of fall.

Ans. X, 3022.5 yds.; ω, 4° 9′.

Example 4.—Given d, 6″; w, 100 lbs.; θ, 3° 23′; initial velocity, 2000 f. s.; find the range and the time of flight.

Ans. X, 3434.3 yds.; T, 6.6478 sec.

Example 5.—Given d, 8″; w, 250 lbs.; θ, 5° 50′; initial velocity, 1700 f. s.; find the range. *Ans.* 4412.7 yds.

Example 6.—Given d, 8″; w, 250 lbs.; θ, 4° 20′; initial velocity, 1700 f. s.; find the range. *Ans.* 3513.5 yds.

CASE IV.

Given the range and the angle of departure, to calculate the initial velocity.

This problem arises when, the initial velocity not having been measured with the chronograph, it is necessary to deduce it from observed results.

We have in this case to find two unknown quantities, V and Z, so as to satisfy equations (1) and (2) of Case I, namely,

$$S_Z = S_V + \frac{X}{C'},$$

$$\frac{A_Z - A_V}{S_Z - S_V} - I_V = \frac{\sin 2\theta}{C'};$$

or, as we may write them, after substituting in the second equation the value of $S_Z - S_V$ from the first,

$$S_Z = S_V + \frac{X}{C'}, \tag{1}$$

$$A_Z - A_V - \frac{X}{C'} I_V = \frac{X}{C'^2} \sin 2\theta. \tag{2}$$

It is necessary to employ the method of trial and error. Assuming a value of V, a corresponding value of Z is determined by (1);

the result of substituting these values in the first member of (2) is then compared with the known value of the second member. If the result is too small, sin 2θ should be smaller in order to satisfy equation (2); that is, θ would have to be smaller to obtain the given range with the assumed initial velocity; therefore V is too great. On the other hand, if the result is too great, V is too small. A new value of V is now assumed in accordance with this indication, and the method of trial and error applied as in Case III.

Example: At the Proving Ground, Annapolis, May, 1888, the mean of five shots with the 250-pound common shell from the 8-inch B. L. R. gave $X = 6002$ yds., $\theta = 8^\circ\ 35'$. The thermometer indicated 83°, the barometer 30″; find the initial velocity.

The computation may be arranged as follows: First finding log δ by the method of Chap. IV, and taking $C' = \frac{C}{\delta}$, since $i = 1$ and $\beta = 1$.

$$\begin{array}{lrl}
 & \log \delta = 9.98058 - 10 & \\
C = \frac{250}{64} & \log C = \underline{0.59176} & \\
 & \log C' = 0.61118 & \\
X = 18006 & \log X = \underline{4.25541} & \\
 & \log \frac{X}{C'} = 3.64423 & \frac{X}{C'} = 4407.9 \\
 & \operatorname{colog} C' = 9.38882 - 10 & \\
2\theta = 17^\circ\ 10' & \log\sin 2\theta = \underline{9.47005} - 10 & \\
 & 2.50310 & \frac{X}{C'^2} \sin 2\theta = 318.49.
\end{array}$$

Assume $V = 1700$, whence

$$\begin{array}{lrr}
S_V = 5701.3 & A_V = 142.13 & I_V = 0.06570 \\
\frac{X}{C'} = \underline{4407.9} & & \log I_V = 8.81757 - 10 \\
S_Z = 10109.2 & \therefore \quad A_Z = \underline{757.57} & \\
 & A_Z - A_V = 615.44 & \log \frac{X}{C'} = \underline{3.64423} \\
 & \frac{X}{C'} I_V = \underline{289.60} & 2.46180 \\
 & 325.84 &
\end{array}$$

The result being too great, V is too small; let us now assume $V = 1720$.

$$\begin{array}{lll}
S_V = 5612.4 & A_V = 136.38 & I_V = 0.06375 \\
\underline{\frac{X}{C'} = 4407.9} & & \log I_V = 8.80448 - 10 \\
S_Z = 10020.3 & \underline{A_Z = 736.28} & \\
 & A_Z - A_V = 599.90 & \underline{\log \frac{X}{C'} = 3.64423} \\
 & \underline{\frac{X}{C'} I_V = 281.00} & \quad .\quad .\quad 2.44871 \\
 & 318.90. &
\end{array}$$

The result is too great; therefore V is still too small. The difference between the results is 6.94, and the error of the last is 0.41; hence the indicated correction to $V = 1720$ is $\frac{41}{694}$ of 20, or 1.2. Let us then try $V = 1721.2$:

$$\begin{array}{lll}
S_V = 5607.2 & A_V = 136.05 & I_V = 0.06363 \\
\underline{\frac{X}{C'} = 4407.9} & & \log I_V = 8.80366 \\
S_Z = 10015.1 & \underline{A_Z = 735.04} & \\
 & A_Z - A_V = 598.99 & \underline{\log \frac{X}{C'} = 3.64423} \\
 & \underline{\frac{X}{C'} I_V = 280.47} & \quad .\quad .\quad 2.44789 \\
 & 318.52. &
\end{array}$$

The result is still too great, but the correction indicated is too small to require recomputation; hence we have for the final result,

$$V = 1721.2.$$

Example 2.—Given d, 3″; w, 7 lbs.; X, 2000 yds.; θ, 8° 29′; find the initial velocity. *Ans.* $V = 935.5$ f. s.

Example 3.—Given d, 10″; w, 500 lbs.; X, 9527 yds.; θ, 12° 47′; find the initial velocity. *Ans.* $V = 1920.9$ f. s.

CASE V.

Given the initial velocity, and the coordinates of a point to be hit, to find the angle of departure, the angle of elevation, etc.

Let s be the inclination of the line joining the point (x, y) of the trajectory to the origin, that is, the angle of sight when the point (x, y) is aimed at; then

$$\frac{y}{x} = \tan s.$$

Since x and y are given, as well as C' and V, we have in this case to find z and θ from the first two of the ballistic formulas, p. 51, which may be written

$$S_z = S_V + \frac{x}{C'}, \tag{1}$$

$$\tan\theta - \tan s = \frac{C' \sec^2\theta}{2}\left[\frac{A_z - A_V}{S_z - S_V} - I_V\right]. \tag{2}$$

Now, α being the angle of elevation, so that $\alpha = \theta - s$,

$$\tan\alpha = \frac{\tan\theta - \tan s}{1 + \tan\theta \tan s}.$$

Therefore, dividing by $1 + \tan\theta \tan s$, equation (2) may be written

$$\tan\alpha = \frac{1 + \tan^2\theta}{1 + \tan\theta \tan s}\,\frac{C'}{2}\left[\frac{A_z - A_V}{S_z - S_V} - I_V\right],$$

or putting

$$\frac{1 + \tan^2\theta}{1 + \tan\theta \tan s} = 1 + \epsilon, \tag{3}$$

$$\tan\alpha = (1 + \epsilon)\,\frac{C'}{2}\left[\frac{A_z - A_V}{S_z - S_V} - I_V\right]. \tag{4}$$

Subtracting unity from each member of (3),

$$\epsilon = \frac{\tan\theta\,(\tan\theta - \tan s)}{1 + \tan\theta \tan s} = \tan\theta \tan\alpha. \tag{5}$$

The value of ϵ is very small, especially when θ and α are small angles; an approximate value of α, say α_1, can therefore be found by neglecting ϵ in equation (4),* and then

$$\theta = \alpha + s$$

gives a corresponding value, θ_1, of θ. Using these approximate values, ϵ is computed by (5), and then a more accurate value of α is determined by equation (4). If the values of α and θ differ greatly from the approximate values, it may be necessary to compute ϵ again and readjust the values of α and θ.

Example: A shell weighing 100 pounds is fired from a six-inch B. L. R. with an initial velocity of 2000 f. s. What must be the angle of elevation to hit a point whose coordinates are $x = 3500$ yds., $y = 500$ yds.?

We have here

$$x = 10500, \qquad C = \frac{100}{36}, \qquad \tan s = \frac{1}{7},$$

whence

$$\frac{x}{C} = 3780, \qquad s = 8^\circ\ 7'\ 48'',$$

and from the table, since $V = 2000$,

$$\begin{array}{lll}
S_V = 4466.2 & A_V = 76.26 & I_V = 0.04224 \\
\dfrac{x}{C} = 3780. & & \\
\hline
S_Z = 8246.2 & A_Z = 400.52 & \\
 & \Delta A = 324.26. &
\end{array}$$

---

* If, in equation (4), we put $\epsilon = \tan^2\alpha$, the approximate formula is

$$\sin 2\alpha = C' \left[\frac{A_z - A_V}{S_z - S_V} - I_V\right],$$

which is the same as the formula for θ, when x is the horizontal range. (Compare equation (2), p. 12.) The result of the hypothesis of the "rigidity of the trajectory" is a somewhat larger value of α, because it is the value of θ corresponding to the actual distance considered as a horizontal range. It is a better first approximation than α_1, when s is positive or, being negative, is small relatively to α and θ. But the latter is sufficient for the purpose of determining ϵ, and is more convenient, for then the value of log tan α used in computing ϵ is the quantity which must in any event be found for use in equation (4). (See the example below.)

When s is negative; if α is numerically smaller than s, so that θ is negative; or if α is numerically larger than s but near it in value, so that θ is relatively small, the result of neglecting ϵ gives a better approximation than the hypothesis of the rigidity of the trajectory.

For tan α_1, the approximate value of tan α, equation (4) gives

$$\log \Delta A = 2.51089$$

$$\log \frac{x}{C} = \log \Delta S = 3.57749$$

$$\log \frac{\Delta A}{\Delta S} = 8.93340 \qquad\qquad \frac{\Delta A}{\Delta S} = 0.08578$$

$$I_V = 0.04224$$

$$\log \left[\frac{\Delta A}{\Delta S} - I_V\right] = 8.63901 - 10 \qquad\qquad \frac{\Delta A}{\Delta S} - I_V = 0.043541$$

$$\log \frac{C}{2} = 0.14267$$

$$\log \tan \alpha_1 = 8.78168 - 10 \qquad\qquad \alpha_1 = 3^\circ\ 27'\ 42''$$

To compute ϵ, $\qquad\qquad s = 8^\circ\ 7'\ 48''$

$$\log \tan \theta_1 = 9.31200 - 10 \qquad\qquad \theta_1 = 11^\circ\ 35'\ 30''$$

$$\log \epsilon = 8.09368 - 10$$

whence $\epsilon = 0.01241$ and $\log (1 + \epsilon) = 0.00535$.

Therefore, for a more accurate value of α we have in equation (4)

$$\log \tan \alpha_1 = 8.78168 - 10$$

$$\log (1 + \epsilon) = 0.00535$$

$$\log \tan \alpha_2 = 8.78703 - 10 \qquad\qquad \alpha_2 = 3^\circ 30'\ 16''.$$

If we recompute ϵ, we shall find

$\epsilon = 0.01261$ and $\log (1 + \epsilon) = 0.00544$;

whence, finally,

$$\log \tan \alpha = 8.78712 - 10 \qquad\qquad \alpha = 3^\circ\ 30'\ 18''$$

$$s = 8^\circ\ 7'\ 48''$$

$$\theta = 11^\circ\ 38'\ 6''.$$

Example 2.—Given d, 6″; w, 100 lbs.; initial velocity, 2000 f. s.; x, 3500 yds.; y, —500 yds.; find the angles of elevation and departure. *Ans.* α, 3° 26′ 40″; θ, —4° 41′ 8″.

Example 3.—Given d, 6″; w, 100 lbs.; initial velocity, 2000 f. s.; x, 3000 yds.; y, 61.85 yds.; find the angle of elevation.

Ans. 2° 49′ 10″.

CHAPTER VI.

MISCELLANEOUS EXAMPLES.

1. Given d, 5″; w, 60 lbs.; I. V., 2000 f. s., and θ, 3°; find the range on the horizontal plane through the muzzle and the angle of fall. Supposing the gun to be mounted 10 feet above the water, apply an approximate correction to the completed range in order to find the range on the water.

Ans. X, 2925 yds.
ω, 3° 56′ 20″.
T, 3.551^s.
Cor., 48 yds.

2. Given d, 10″; w, 500 lbs.; I. V., 2000 f. s., and θ, 2°; find the coördinates of the point at which $\varphi = 0$.

Ans. X_0, 1308.2 yds.
Y_0, 23.7 yds.

3. Given d, 5″; I. V., 2000 f. s.; w, 60 lbs.; X, 3000 yds.; find θ and ω.

Ans. θ, 2° 58′
ω, 4° 6′

4. Given d, 5″; I. V., 2000 f. s.; w, 60 lbs.; θ, 4°; find x_0 and Y.

Ans. x_0, 2028.9 yds.
Y, 78.79 yds.

5. Given d, 5″; I. V., 2000 f. s.; w, 60 lbs.; θ, 4° 22′; find X and T.

Ans. X, 3919.3 yds.
T, 8.222^s.

6. Given for the Hotchkiss 6-pdr. R. F. gun d, 2.24″; w, 6 lbs.; I. V., 1818 f. s.; X, 3000 yds.; jump, + 2′ 10″; i, coefficient of form, 0.9; find θ, ω and the approximate dangerous space, at the given range, for an object 20 feet high.

Ans. θ, 4° 42′ 48″.
ω, 7° 12′ 50″.
D. S., 52.7 yds.

7. Given d, 8″; w, 250 lbs.; I. V., 2000 f. s.; θ, 10°; find x_0 and Y.

Ans. x_0, 4401 yds.
Y, 445.5 yds.

8. Given d, 8″; w, 250 lbs.; X, 3228 yds.; θ, 3° 55′; find the initial velocity. (I. V. about 1700 f. s.) *Ans.* I. V., 1695.3 f. s.

9. The Hotchkiss 6-pdr. common shell has an ogival head of $2\frac{1}{2}$ calibres radius, and the correction for shape of heads is 0.9. Given for this gun d, 2.24″; w, 6 lbs.; I. V., 1800 f. s.; θ, 4°; find the coordinates of the vertex. *Ans.* x_0, 1470 yds.
Y, 58.63 yds.

10. Given d, 10″; w, 500 lbs.; I. V., 2000 f. s.; T, 3.2^s; X, 2000 yds.; find the angle of fall, the *approximate* height of the vertex and the *approximate* dangerous space for a ship's broadside 21 ft. high.
Ans. Y, 41.216 ft.
ω, 1° 42′ 59″.
D. S., 233.59 ft.

11. Given d, 6″; w, 100 lbs.; I. V., 2000 f. s.; permanent angle, 0° 54′; distance between sights, 66.81″; jump, + 4′; X, 3000 yds.; use Siacci's formulas and find the angle of elevation, the time of flight and the sight-bar height for the given range.
Ans. θ, 2° 49′ 02″.
α, 2° 45′ 02″.
T, 5.6178 .
H, 3.2094″.

12. Given, for the 10″ B. L. R., I. V., 2000 f. s.; w, 500 lbs.; jump, + 5′; R, 10,000 yards; find θ, α, T, ω, z', v the true final velocity, the approximate height of the maximum ordinate and the dangerous space for a target 21 feet high, the gun being 9 feet above the water.
Ans. θ, 12° 47′ 27″. T, 22.878^s. S, 19.636 yds.
α, 12° 42′ 27″. v, 1047.2 f. s. Y, 702.2 yds.
ω, 19° 37′ 24″. z', 1011.5 f. s.

13. Given d, 5″; w, 60 lbs.; I. V., 2000 f. s., and θ, 1°; find the time of flight, range and angle of fall, approximately, and the dangerous space for a target 21 feet high, the gun being mounted 9 feet above the water. *Ans.* T, 2.168^s. ω, 1° 14′ 16″.
X, 1296 yds. S, 335.54 yds.

14. Given d, 12″; w, 850 lbs.; I. V., 2100 f. s.; X, 14,000 yds.; calculate θ, ω, T and the final velocity. *Ans.* θ, 18° 57′ 57″.
ω, 29° 40′ 20″.
T, 33.68^s.
f. v., 1052 f. s.

15. Given d, 12″; w, 850 lbs.; I. V., 2100 f. s.; θ, 18° 57′ 57″; calculate x_0, Y, v_0 and t_0.

Ans. x_0, 7912 yds.
v_0, 1176 f. s.
Y, 1594 yds.
t_0, 15.742^s.

16. The velocity of a 12″ shell, w 850 lbs., at the vertex of its trajectory, is 1176 f. s., and the angle of fall is — 29° 40′ 20″; calculate the distance from the foot of the ordinate of the vertex to the point of fall and the corresponding time of flight.

Ans. Dist., 6275.3 yds.
T, 18.316^s.

17. Given I. V., 2000 f. s.; d, 6″; w, 100 lbs.; R, 2000 yds. on the horizontal plane through the gun; find θ and ω. Find an approximate correction to be applied to the given range in order to determine what the range would be on a horizontal plane 18 feet below the gun.

Ans. θ, 1° 41′ 20″.
ω, 2° 02′ 15″.
Cor., + 164 yds.

18. The observed range of a 12″ shell, w, 850 lbs.; I. V., 2100 f. s.; thermometer, 52°; barometer, 30.60, being 5000 yards, calculate what the range would be under the standard atmospheric conditions of the ballistic tables.

Ans. $X = 5034$ yds.

19. Given d, 12″; w, 850 lbs.; I. V., 2100 f. s.; θ, 0° 39′ 38″; X, 1000 yds.; calculate the final velocity, the time of flight and the *approximate* height of the vertex. In this case, θ and ω being small angles, the final velocity will not differ appreciably from Z. Calculate, also, the penetration of the shell, at the given range, in steel armor. The formula for the penetration in inches, ϵ, in *thick* plates of Creusot steel is

$$\epsilon^{\frac{7}{10}} = [6.99066 - 10]\,\frac{w^{\frac{1}{2}}v}{d^{\frac{3}{4}}}\,.$$

The quantity in brackets is the logarithm of a constant, w the weight in pounds, v the velocity in feet per second, and d the diameter in inches.

Ans. f. v., 1964 f. s.
T, 1.478^s.
Y, 8.7666′.
ω, 0° 47′ 37″.
ϵ, 21.958″.

CHAPTER VII.

CORRECTION OF THE FIRE OF GUNS FOR WIND, AND MOTION OF GUN AND TARGET.

General Discussion.—Among the principal causes of variation in range and deviation in direction of flight of rifled projectiles, independent of the errors of the gun itself, are the influence of the wind and the motion of the gun. The motion of the target, if not allowed for, also affects accuracy of fire.

The errors due to the above causes are corrected when the amount of the errors is once determined by setting the sliding leaf of the sight-bar in the direction in accordance with the rule given in Chapter I.

The divisions on the sliding leaf of the navy sights are marked in knots, the distances to the right or left of the center mark being computed for a fixed range of 2000 yards, to correct the deviations which would occur at that range for the combined speeds *producing deviation* of the gun and target at right angles to the line of fire. For other ranges, the time of flight being not exactly proportional to the range, the correction made will be not quite exact but very approximate. In practice it is usual to estimate the effect of the wind, in divisions on the leaf, and to set it accordingly for the first trial shot—correcting the setting for the next shot—but it would be of advantage if a rough table was constructed showing the effect of the wind at various ranges for different velocities of the wind.

Hélie's Formulas for the Effect of the Wind.—To compute approximately the effect of the wind we have the following formulas, based on the assumption that the deviation in range and laterally is proportional to the velocity of the wind in the two directions respectively, neglecting the areas of cross section of the projectile and taking account of the *time* the projectile is in the air.

In vacuo, the horizontal component of the velocity being $v \cos \theta$, the range would be $tv \cos \theta$, and if in air the range is R. The difference $tv \cos \theta - R$ is due to the resistance of the air—or it may be considered as due to a gradually diminishing wind—of which the original velocity is $v \cos \theta$; then we have, calling β the angle made by the direction of the wind with the line of fire, the following proportion:

$$\frac{d}{w \sin \beta} = \frac{vt \cos \theta - R}{v \cos \theta};$$

or

$$d = w \sin \beta \left(t - \frac{R}{v \cos \theta} \right)$$

for the deviation laterally, and

$$d' = w \cos \beta \left(t - \frac{R}{v \cos \theta} \right)$$

for the deviation in range. The equations give approximate results only.

The Motion of the Gun.—In this case the projectile as it leaves the gun is supposed to have a velocity equal to and in the direction of the motion of the gun itself. Now if we can consider the motion in this direction as practically unresisted by the air, an approximate solution is very easily reached.

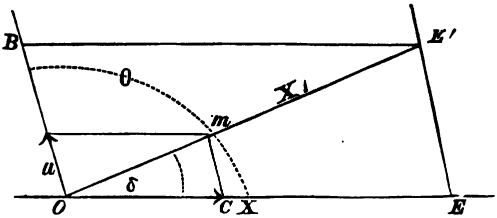

FIG. 1.

On the above hypothesis consider Fig. 1 as the horizontal plane through the point O at which the gun is situated. Suppose the gun to be moving toward B with a velocity U, and if OE is the line of fire, E being the point of fall of the projectile with an initial velocity V and angle of elevation α when the gun is stationary. And if X is the range and T the time of flight corresponding to these data, and if the projectile during the time T has a uniform velocity in the direction of OB equal to U, then at the end of the time T it will fall at E'; EE' being equal and parallel to $OB = UT$.

The time T and range X can be found from formulae in which they occur as functions of $V.\alpha$, or may be taken from range tables.

For ranges, then, at angles of elevation less than 5°, we may with close approximation take T for a range X that would be given for an initial velocity V and angle of elevation α.

Thus approximating, we have from the triangle $BE'O$, the line $BE'=X$,

$$X_1^2=X_2^2+U^2T^2+2UTX\cos\theta.$$

But U^2T^2 is very small when compared with X_2^2, and may be neglected, or multiplied by a quantity less than unity, $\cos^2\theta$ say, and we have

$$X_1=X+UT\cos\theta. \qquad (1)$$

From the similar triangles OCm and OEE' we have

$$\frac{\sin\delta}{\sin(\theta-\delta)}=\frac{U}{V\cos\alpha}=\frac{UT}{X},$$

or $X\sin\delta=UT\sin(\theta-\delta)$.

Since δ is always a very small angle and vanishes when $\theta=0$, we may write $\sin\theta$ for $\sin(\theta-\delta)$. Again, the sine and tangent of δ are sensibly equal, and if the lateral deviation is $(X\tan\delta)$, we have

$$d=UT\sin\theta. \qquad (2)$$

From formulas (1) and (2) we see that the sign of the second term of the second member of (1) is positive for values of θ between 0° and 90°, and negative for values between 90° and 180°; θ being taken as a positive angle and reckoned from the line of fire on either side around to the rear of the gun. d is always positive.

The Motion of the Target.—Let Fig. 2 be the horizontal plane through OS.

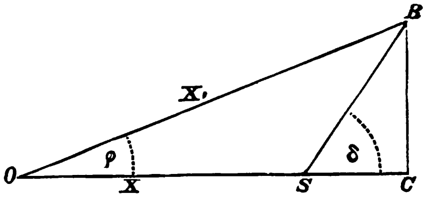

FIG. 2.

If the target is at S, Fig. 2, and moving with a velocity U toward B, the line SB making an angle δ with the line of fire OS, the gun being fixed at O. At the end of the time T (T being the time for a shot to travel the range X or OS), the target will have arrived at some point E, and $BS = UT$.

Now the gun must be so aimed that the shot will fall at B at the moment the target arrives at that point, hence the sight bar must be set for a range X_1 and the sliding leaf to allow for a lateral deviation BC at the range X_1. We have $BC = UT \sin \delta$, and $Sc = UT \cos \delta$; and since φ is always a small angle, we have approximately

$$X_1 = X + UT \cos \delta, \tag{3}$$

$$d = UT \sin \theta. \tag{4}$$

If d' is the lateral deviation for the range X, it will be found practically to differ but little from d, and may be used in its stead.

d is always positive, and the second term of the second member of (3) is positive for values of S between 0° and 90°, and negative for values between 90° and 180°, δ being taken as a positive angle and reckoned from the line of fire on either side around to the rear of the gun.

CHAPTER VIII.

ON THE PREPARATION AND USE OF RANGE TABLES.

General Discussion.—A range table should contain all the information which is necessary in order to use effectively any given projectile. Since, in the use of naval guns, the angle of sight (see Chapter I for definition of term) is generally small, the trajectory may be assumed sensibly rigid; or, in other words, the sight bar may be used without regard to what may be the (small) elevation or depression of the target.

If the ranges are to be computed for various angles of departure, by the methods indicated in Chapter V, it is evident that the jump angle must be known; since without this the angles of elevation cannot be found at all. This being known, successive values of the angle of departure 1°, 2°, 3°, etc., are assumed, and the ranges, times of flight, greatest ordinates, and angles of fall corresponding are found. This method will give good results for projectiles of a form similar to those from which the tables were made; but, if the cost of firing a gun a large number of times is not too great, or if some other question of convenience does not forbid it, it is better to form the range table by experiment.

In order that a range table may be of value, it must tell the range, time of flight, etc., when referred to the proper plane. The best *plane of reference* is a matter of choice, but the horizontal plane through the muzzle or that through the trunnions is usually taken.

Data a complete Range Table should contain.—At the top of the table, and not in the columns, should be found the following data: *Calibre of gun, weight of projectile, radius of gun, weight of charge, kind of powder, initial velocity, angle of jump, kind of carriage, permanent angle,* and it should be stated whether the table has been determined by experiment or by computation.

In the columns from left to right should be found the following: *Range in yards* (every 100 yards), *angle of elevation, time of flight, angle of fall, remaining velocity, mean error in range, mean error in direction, mean vertical error, penetration in steel armor.*

These data are absolutely necessary. The following are useful and should be given if obtainable: The per cent. of change of range in yards due to an increase or decrese of initial velocity by 100 f. s.; the variation in range at each range allowable in order to hit an object of average height of ships, say 20 feet, forming a column by itself.

To Range a Gun by Experiment.—This is done by setting the axis of the bore by level or by quadrant to certain well chosen angles of elevation, and the gun being fired, the ranges and times of flight are measured. These quantities must then be referred to the horizontal plane of reference before mentioned, which is usually that through the trunnions. The shots should be fired under identical conditions, so far as is possible, accurate records being kept of the barometer and of the wet and dry bulb thermometers. The ranges and times of flight are then reduced to what they would have been had the density of the air been that of the standard adopted. The charges and projectiles are accurately weighed, and an accurate record of the range for each shot carefully retained. Several shot are fired at each angle, not less than five, and from the records sufficient data are obtainable to supply all that is necessary to compute the gun's probable rectangle, or the mean errors in range and direction for each range.

Suppose that on the proving ground several successive shots at each elevation have given the following results; these being corrected for the jump, effect of the wind, and density of the air:

Mean Angles of Departure.	Mean Ranges.
1° 48′	1048
2° 48′	1590
5° 48′	2465
7° 48′	3292
10° 48′	4108
15° 48′	5245

In order to prepare a range table from this data, we may proceed as follows. Draw rectangular axes: take that of X for the ranges, using the scale, say 1 inch = 500 feet; and that of Y for the angles of departure, using 1 inch = 1°, say. Lay off the points and draw a smooth curve through them. This curve will pass through the origin, making with the axis of abscissae an angle whose tangent is

$\frac{g}{2v^2} \cdot \frac{180}{\pi} \cdot \frac{n}{m}$, n being the magnitude adopted to represent 1°, m that which represents 1 foot, and v is the muzzle velocity. The curve will be convex towards the axis of ranges, its curvature will increase as we go from the origin until it becomes tangent to the ordinate which corresponds to the maximum range.

From the curve so drawn, take off the angles of departure corresponding to the ranges in increments of 500 yards, thus forming the following table:

Ranges.	Angles.	Δ	Δ^2
0	0		
		44	
500	44′		11
		55	
1000	99′		10
		65	
1500	164′		8
		73	
2000	237′		9
		82	
2500	319′		7
		89	
3000	408′		

From this we find (see Ray's Algebra, Art. 335), calling a_{1600}, a_{1700}, etc., the angles of departure at 1600, 1700, . . . , yards,

$$a_{1600} = 164 + \frac{1}{5}\left(73 - \frac{4}{10} \cdot 8\right) = 177.88',$$

$$a_{1700} = 164 + \frac{2}{5}\left(73 - \frac{3}{10} \cdot 9\right) = 192.12'.$$

and so on. It is not desirable to retain anything beyond the nearest minute.

Range Table by Experiment—Second Method.—To this end the gun would be laid a number of times at successive angles of inclination by a spirit level and fired; and the mean ranges found be called the range due to this angle. If the firing take place over the land, the points of fall would be found accurately by measurement; but if it takes place over water, these points must be observed by two plane tables, one at the gun and the other at a point which will give the best cut. In either case, the planes on which the ranges are measured would probably not be the horizontal plane through the gun, the plane to which the ranges must be referred; and we must apply to the angle at which the gun was laid by spirit level, a correction such that the resulting angle shall be the angle of elevation. In the figure, let A be the position of the gun at height h above

BT, the plane on which the range *R* is measured; let *T* be the point struck, and *B* be vertically under *A*. It is to be remembered that in all actual cases *R* is very great compared with *h*. The angle

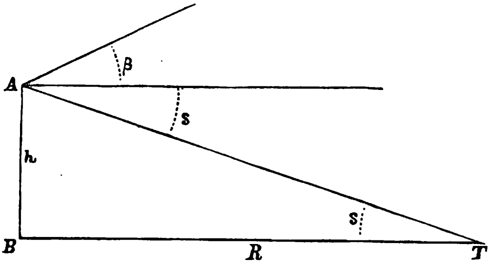

FIG. 3.

β would be that given by the spirit level; and since *AT* is the line of sight, $\beta + s$ is the angle of elevation. Also

$$\tan s = \frac{h}{R}.$$

The observation is then recorded that, with an angle of elevation $\beta + s$, a range *R* was found. Obviously, if we slowly raise the sight-bar, if one had been fitted before the gun was ranged, until we see the point *T* across the sights, we may put a scratch across its face and write the value of *R* on it. As, however, the bars are usually marked in even hundreds of yards, this would not be the best way to proceed.

If possible, the experimental firing should take place in standard conditions of barometer and thermometer, and the ranges and deflections corrected for the effect of the wind.

Suppose, then, that a number of shots had been fired within the limits within which it was intended to establish the range table, proceeding by increments of 2° say, and that these had been corrected. In order to find the permanent angle *i* (see figure in Chapter I), it is usual to take the deflections at some considerable angle of elevation, to compute its value (see figure already mentioned). If it should be found that the permanent angle so computed at a selected angle does not properly correct for the observed deflections at other angles, the sight bar might be left vertical, and a table constructed showing the amount of lateral motion to give the sight notch at each angle.

There will remain, since the correct determination of the permanent angle will cause the shot always to fall in the vertical plane containing the line of sight, a series of observations connecting angles of elevation $\beta + s$ (the angle s is sometimes called the *additional* angle) with range. Since, in the general case, R is sensibly equal to $\sqrt{R^2 + h^2}$, we may record the former as the range. Also, in the sight-bar triangle (*ABC*, Chapter I), the angle *BAC* is the angle of elevation; and since *AB* is known, being the distance between sights, we may, having any number of relations connecting range and elevation, find the corresponding ones connecting range and sight-bar height. In case the bar is set at a permanent angle i, we may obviously (see figure, Chapter I) mark the line BC' which represents the sight-bar in this case.

We may then, to mark the sight-bar and determine the range table completely from the data we have, take either angles of elevation and ranges or sight-bar heights and ranges. If we take the former, we might proceed as described in the first method. Or else, assuming a proper analytical form for the law connecting angles of elevation and range, compute the constants in the assumed equation by the Method of Least Squares. The latter method, when carefully chosen, probably involves less labor than the former, and is certainly more satisfactory and accurate.

The form of the relation connecting angles of elevation and range is not known. In a non-resisting medium it is

$$\sin 2\alpha = \frac{g}{V^2} R;$$

and this indicates that we might hope to get good results by assuming such form as

$$\sin 2\alpha = aR + bR^2;$$

since the true form is doubtless more complex. For small angles (up to 5° or 6°) a more convenient form for computation would be

$$\alpha = aR + bR^2.$$

In the use of this, by taking α in minutes of arc, it would be a fairly large quantity, and a and b would not be so small as they would be if we took $\sin \cdot 2\alpha$. In case there is jump, we may then finally adopt the form

$$\theta = bR + cR^2,$$

where θ is the angle of departure.

Therefore, writing in the form

$$\frac{\theta}{R} = b + cR, \qquad (1)$$

we have only b and c to determine by least squares.

EXAMPLE.

DATA FOR RANGING THE 6-PDR. HOTCHKISS R. F. GUN.

Place, Naval Ordnance Proving Ground, Annapolis, Md.; *I. W.*, 1804 f. s.; *jump*, +2′ 10″; *w*, 6 lbs.; *d*, 2.24″; *i*, 0.9; log $\frac{C}{i}$ 0.12341; *height of gun above water*, 24.5 feet; *radius of gun*, 37.16″; *permanent angle*, 2°.

No. of Group.	No. of Shots in Group.	Mean Range on Water.	Quadrant Angle.	Time of Flight.	Wind.		Barom.	Therm.
					Direction	Force.		
		yds.		sec.	↓	f. s.		
1	5	422.2	—0° 45′ 00″	0.8	$\beta = 0$	6.1	30.28	78°
2	10	683.8	0 0 0	1.3		calm	30.25	76°
3	3	1285.3	1° 0 0	2.7	"	2.0	30.25	84°
4	5	2315.0	3° 0 0	5.3		calm	30.25	76°
5	5	2780.4	4° 0 0	7.0	"	5.1	30.29	81°
6	5	3183.6	5° 1′ 10″	8.3	"	4.2	30.30	82°
7	4	3936.0	7° 0 0	11.0	"	4.2	30.30	84°
8	5	4595.0	9° 0 0	13.3	"	4.3	30.30	84°

To construct a range table for this gun, using the above data, but for an initial velocity of 1818 f. s. instead of that used in the firing, we proceed as follows:

1. Find s and θ; $\theta =$ Quad. Angle $+ j + s$.
2. Correct the observed range for δ, the variation of the barometer and thermometer from the standard (see Chap. IV).

3. Correct the observed range for the influence of the wind.

4. With range corrected for wind and δ, compute θ for I. V. 1818 f. s., and also for 1804 f. s. The difference of the two angles gives a correction to be applied to θ to find what that angle would have been had the I. V. been 1818 f. s. instead of 1804 f. s.

5. With the corrected values of θ for each range, and using the formula $\frac{\theta}{R} = b + cR$; determine b and c by least squares.

Following this method in the above case, the following equation connecting θ and R was deduced; θ being given in minutes of arc.

$$\theta = 0.048535R + 0.000015309R^2. \qquad (2)$$

Using this equation for any range X, the corresponding angle θ was deduced. Applying the angle of jump, the angle of elevation was found, and thence the sight-bar was marked for the ranges at intervals of 100 yards.

To find the *angle of fall*, we proceed as follows: If ω is the angle of fall for any range R; or in other words, if the trajectory be assumed rigid, ω is the angle between the radius vector to the curve and the tangent, we have

$$\tan\omega = \frac{Rd\theta}{dR}.$$

(See Rice and Johnson's Calculus, Art. 317.)

Then, from equation 1, in units of circular measure, we have

$$\tan\omega = \frac{Rd\theta}{dR} = \sin 1'bR + 2\sin 1'cR^2. \qquad (3)$$

In this particular case,

$$\tan\omega = 0.0^4 14119R + 0.0^8 89068R^2,$$

from which the angle of fall ω was computed for the range R.

Thus the equations derived give a quick and easy method of finding the angles of elevation and projection and angles of fall. The time could similarly be found for all intermediate points; the curve drawn between time of flight and angles of elevation is evidently nearly straight. There is only one assumption made which requires justification; this is that the range on any line is independent of the inclination of the line to the horizontal. For small inclinations, and when the attainable accuracy in all respects is considered, it is evident that we shall come as near the truth by this means as any other now available.

For the 8″ gun, mark II, I. V. 1700 f. s., jump 0°,

$$\theta = 0.055754R + 0.0000051222R^2,$$

$$\tan \omega = 0.0^4 16218R + 0.0^8 298R^2.$$

To find the Value of i, the Coefficient of Form, from the Results of Actual Firing, with a Projectile differing from the Standard.

Example.—The data is as follows:

X, corrected for wind $= 2785$ yards;
barometer, 30.29; thermometer, 81° F.;
$\log \delta = 9.98658 - 10$; $\log \frac{\delta}{C} = 9.90893 - 10$;
I. V. $= 1804$ f. s.; radius of head of projectile, 3 calibres.

We have the equations (1) and (2), Case IV,

$$A_V + I_V X \frac{1}{C'} + X \sin 2\theta \frac{1}{C'^2} = A_Z, \tag{1}$$

$$\frac{S_Z - S_V}{X} = \frac{1}{C'}. \tag{2}$$

Assume a value of $\frac{1}{C'} = \frac{\delta}{C}$; find A_Z from (1) and thence S_Z, if (2) is satisfied, then we have the correct value of $\frac{1}{C'}$; otherwise take the value obtained from (2) as a second approximation. This approximation is very slow, but in practice may be hastened.

In this manner was found from the above data,

$$\log \frac{1}{C'} = 9.86610 - 10 = \log \frac{\delta i \beta}{C}; \quad \therefore \quad i\beta = .90611.$$

But β for the given range being very close to unity, $i\beta$ may be assumed $= i$, $\therefore$ $i = 0.9$ as a general value, $\log \frac{C}{i} = 0.12341$ for this projectile.

BALLISTIC TABLES

FOR

54074

STANDARD NAVY PROJECTILES

WITH

OGIVAL HEADS OF TWO CALIBERS RADIUS.

COMPUTED BY

R. R. INGERSOLL,

LIEUTENANT COMMANDER, UNITED STATES NAVY,

FROM

MAYEVSKI'S LAWS OF THE RESISTANCE OF THE AIR, AS DEDUCED FROM
KRUPP'S EXPERIMENTS.

1893.

WASHINGTON:
GOVERNMENT PRINTING OFFICE,
1893.

TABLE I.

V	S_v	*Diff.*	A_v	*Diff.*	I_v	*Diff.*	T_v	*Diff.*
3600	000.0	21.1	0.00	0.00	0.00000	10	0.000	6
3590	21.1	21.1	0.00	0.00	0.00010	10	0.006	6
3580	42.2	21.2	0.00	0.01	0.00020	11	0.012	6
3570	63.4	21.3	0.01	0.01	0.00031	11	0.018	6
3560	84.7	21.5	0.02	0.01	0.00042	11	0.024	6
3550	106.2	21.5	0.03	0.01	0.00053	11	0.030	6
3540	127.7	21.5	0.04	0.02	0.00064	11	0.036	6
3530	149.2	21.5	0.06	0.02	0.00075	11	0.042	6
3520	170.7	21.5	0.08	0.02	0.00086	11	0.048	6
3510	192.2	21.6	0.10	0.02	0.00097	12	0.054	6
3500	213.8	21.7	0.12	0.03	0.00109	11	0.060	6
3490	235.5	21.9	0.15	0.03	0.00120	12	0.066	6
3480	257.4	22.0	0.18	0.03	0.00132	11	0.072	6
3470	279.4	22.0	0.21	0.03	0.00143	12	0.078	7
3460	301.4	22.0	0.24	0.04	0.00155	12	0.085	6
3450	323.4	22.0	0.28	0.04	0.00167	12	0.091	7
3440	345.4	22.1	0.32	0.04	0.00179	12	0.098	6
3430	367.5	22.1	0.36	0.04	0.00191	12	0.104	7
3420	389.6	22.3	0.40	0.04	0.00203	12	0.111	6
3410	411.9	22.4	0.44	0.05	0.00215	13	0.117	7
3400	434.3	22.5	0.49	0.05	0.00228	12	0.124	7
3390	456.8	22.5	0.54	0.06	0.00240	13	0.131	6
3380	479.3	22.5	0.60	0.06	0.00253	13	0.137	7
3370	501.8	22.5	0.66	0.06	0.00266	13	0.144	6
3360	524.3	22.7	0.72	0.06	0.00279	13	0.150	7
3350	547.0	22.7	0.78	0.07	0.00292	13	0.157	7
3340	569.7	22.8	0.85	0.07	0.00305	13	0.164	7
3330	592.5	22.8	0.92	0.07	0.00318	13	0.171	7
3320	615.3	22.9	0.99	0.08	0.00331	13	0.178	7
3310	638.2	22.9	1.07	0.08	0.00344	14	0.185	7
3300	661.1	23.1	1.15	0.09	0.00358	14	0.192	7
3290	684.2	23.2	1.24	0.09	0.00372	14	0.199	7
3280	707.4	23.2	1.33	0.09	0.00386	14	0.206	7
3270	730.6	23.2	1.42	0.09	0.00400	14	0.213	7
3260	753.8	23.3	1.51	0.10	0.00414	14	0.220	7
3250	777.1	23.5	1.61	0.10	0.00428	14	0.227	7
3240	800.6	23.5	1.71	0.10	0.00442	14	0.234	8
3230	824.1	23.6	1.81	0.11	0.00456	15	0.242	7
3220	847.7	23.6	1.92	0.11	0.00471	15	0.249	7
3210	871.3	23.6	2.03	0.12	0.00486	15	0.256	8

TABLE I—CONTINUED.

V	S_v	*Diff.*	A_v	*Diff.*	I_v	*Diff.*	T_v	*Diff.*
3200	894.9	23.8	2.15	0.12	0.00501	15	0.264	7
3190	918.7	23.9	2.27	0.13	0.00516	15	0.271	8
3180	942.6	24.0	2.40	0.13	0.00531	15	0.279	7
3170	966.6	24.0	2.53	0.14	0.00546	15	0.286	7
3160	990.6	24.0	2.67	0.14	0.00561	16	0.293	8
3150	1014.6	24.1	2.81	0.14	0.00577	16	0.301	7
3140	1038.7	24.2	2.95	0.14	0.00593	16	0.308	8
3130	1062.9	24.3	3.09	0.15	0.00609	16	0.316	8
3120	1087.2	24.4	3.24	0.15	0.00625	16	0.324	8
3110	1111.6	24.5	3.39	0.16	0.00641	16	0.332	8
3000	1136.1	24.6	3.55	0.16	0.00657	16	0.340	8
3090	1160.7	24.7	3.71	0.17	0.00673	17	0.348	8
3080	1185.4	24.7	3.88	0.17	0.00690	17	0.356	9
3070	1210.1	24.7	4.05	0.18	0.00707	17	0.365	8
3060	1234.8	24.8	4.23	0.19	0.00724	17	0.373	8
3050	1259.6	24.9	4.42	0.19	0.00741	17	0.381	8
3040	1284.5	25.0	4.61	0.19	0.00758	18	0.389	8
3030	1309.5	25.1	4.80	0.20	0.00776	18	0.397	9
3020	1334.6	25.2	5.00	0.20	0.00794	18	0.406	8
3010	1359.8	25.3	5.20	0.20	0.00812	18	0.414	8
3000	1385.1	25.4	5.40	0.21	0.00830	18	0.422	8
2990	1410.5	25.5	5.61	0.22	0.00848	18	0.430	9
2980	1436.0	25.5	5.83	0.23	0.00866	18	0.439	8
2970	1461.5	25.6	6.06	0.23	0.00884	19	0.447	9
2960	1487.1	25.7	6.29	0.23	0.00903	19	0.456	9
2950	1512.8	25.8	6.52	0.24	0.00922	19	0.465	8
2940	1538.6	25.9	6.76	0.25	0.00941	19	0.473	9
2930	1564.5	26.0	7.01	0.26	0.00960	20	0.482	9
2920	1590.5	26.1	7.27	0.26	0.00980	20	0.491	9
2910	1616.6	26.1	7.53	0.27	0.01000	20	0.500	9
2900	1642.7	26.2	7.80	0.27	0.01020	20	0.509	9
2890	1668.9	26.4	8.07	0.28	0.01040	20	0.518	9
2880	1695.3	26.5	8.35	0.28	0.01060	21	0.527	9
2870	1721.8	26.5	8.63	0.29	0.01081	21	0.536	9
2860	1748.3	26.6	8.92	0.30	0.01102	21	0.545	10
2850	1774.9	26.7	9.22	0.30	0.01123	21	0.555	10
2840	1801.6	26.8	9.52	0.31	0.01144	21	0.565	9
2830	1828.4	26.9	9.83	0.31	0.01165	22	0.574	10
2820	1855.3	27.0	10.14	0.32	0.01187	22	0.584	9
2810	1882.3	27.1	10.46	0.32	0.01209	22	0.593	10

TABLE I—CONTINUED.

V	S_v	*Diff.*	A_v	*Diff.*	I_v	*Diff.*	T_v	*Diff.*
2800	1909.4	27.2	10.78	0.33	0.01231	22	0.603	9
2790	1936.6	27.2	11.11	0.34	0.01253	23	0.612	10
2780	1963.8	27.4	11.45	0.35	0.01276	23	0.622	10
2770	1991.2	27.5	11.80	0.37	0.01299	23	0.632	10
2760	2018.7	27.6	12.17	0.38	0.01322	23	0.642	10
2750	2046.3	27.7	12.55	0.38	0.01345	24	0.652	10
2740	2074.0	27.8	12.93	0.39	0.01369	24	0.662	11
2730	2101.8	27.9	13.32	0.39	0.01393	24	0.673	10
2720	2129.7	28.0	13.71	0.40	0.01417	24	0.683	10
2710	2157.7	28.0	14.11	0.40	0.01441	25	0.693	11
2700	2185.7	28.2	14.51	0.41	0.01466	25	0.704	10
2690	2213.9	28.3	14.92	0.42	0.01491	25	0.714	11
2680	2242.2	28.4	15.34	0.43	0.01516	26	0.725	10
2670	2270.6	28.5	15.77	0.45	0.01542	26	0.735	11
2660	2299.1	28.6	16.22	0.46	0.01568	26	0.746	11
2650	2327.7	28.7	16.68	0.47	0.01594	27	0.757	11
2640	2356.4	28.8	17.15	0.47	0.01621	27	0.768	11
2630	2385.2	29.0	17.62	0.48	0.01648	27	0.779	11
2620	2414.2	29.1	18.10	0.49	0.01675	27	0.790	11
2610	2443.3	29.2	18.59	0.50	0.01702	27	0.801	11
2600	2472.5	29.3	19.09	0.51	0.01729	28	0.812	11
2590	2501.8	29.4	19.60	0.52	0.01757	28	0.823	11
2580	2531.2	29.5	20.12	0.53	0.01785	29	0.834	12
2570	2560.7	29.6	20.65	0.55	0.01814	29	0.846	11
2560	2590.3	29.8	21.20	0.55	0.01843	29	0.857	12
2550	2620.1	29.9	21.75	0.56	0.01872	30	0.869	11
2540	2650.0	30.0	22.31	0.57	0.01902	30	0.881	12
2530	2680.0	30.0	22.88	0.59	0.01932	30	0.893	12
2520	2710.0	30.2	23.47	0.60	0.01962	31	0.905	12
2510	2740.2	30.3	24.07	0.61	0.01993	31	0.917	12
2500	2770.5	3.0	24.68	0.06	0.02024	3	0.929	1
2499	2773.5	3.1	24.74	0.06	0.02027	3	0.930	1
2498	2776.6	3.0	24.80	0.06	0.02030	3	0.931	1
2497	2779.6	3.1	24.86	0.06	0.02033	3	0.932	2
2496	2782.7	3.0	24.92	0.06	0.02036	3	0.934	1
2495	2785.7	3.1	24.98	0.06	0.02039	4	0.935	1
2494	2788.8	3.0	25.04	0.07	0.02043	3	0.936	1
2493	2791.8	3.1	25.11	0.06	0.02046	3	0.937	1
2492	2794.9	3.0	25.17	0.06	0.02049	3	0.938	2
2491	2797.9	3.1	25.23	0.07	0.02052	3	0.940	1

TABLE I—Continued.

V	S_v	*Diff.*	A_v	*Diff.*	I_v	*Diff.*	T_v	*Diff.*
2490	2801.0	3.0	25.30	0.06	0.02055	3	0.941	1
2489	2804.0	3.1	25.36	0.06	0.02058	3	0.942	1
2488	2807.1	3.1	25.42	0.06	0.02061	3	0.943	2
2487	2810.2	3.0	25.48	0.07	0.02064	4	0.945	1
2486	2813.2	3.1	25.55	0.06	0.02068	3	0.946	1
2485	2816.3	3.0	25.61	0.06	0.02071	3	0.947	2
2484	2819.3	3.1	25.67	0.07	0.02074	3	0.949	1
2483	2822.4	3.0	25.74	0.06	0.02077	4	0.950	1
2482	2825.4	3.1	25.80	0.06	0.02081	3	0.951	2
2481	2828.5	3.1	25.86	0.07	0.02084	3	0.953	1
2480	2831.6	3.0	25.93	0.06	0.02087	3	0.954	1
2479	2834.6	3.1	25.99	0.07	0.02090	3	0.955	1
2478	2837.7	3.1	26.06	0.06	0.02093	3	0.956	1
2477	2840.8	3.1	26.12	0.07	0.02096	3	0.957	2
2476	2843.9	3.1	26.19	0.06	0.02099	4	0.959	1
2475	2847.0	3.1	26.25	0.07	0.02103	3	0.960	1
2474	2850.1	3.1	26.32	0.06	0.02106	3	0.961	1
2473	2853.2	3.1	26.38	0.07	0.02109	3	0.962	2
2472	2856.3	3.0	26.45	0.06	0.02112	4	0.964	1
2471	2859.3	3.1	26.51	0.07	0.02116	3	0.965	1
2470	2862.4	3.1	26.58	0.06	0.02119	3	0.966	1
2469	2865.5	3.0	26.64	0.07	0.02122	3	0.967	1
2468	2868.5	3.1	26.71	0.07	0.02125	3	0.968	2
2467	2871.6	3.0	26.78	0.06	0.02128	4	0.970	1
2466	2874.6	3.1	26.84	0.07	0.02132	3	0.971	1
2465	2877.7	3.1	26.91	0.07	0.02135	3	0.972	2
2464	2680.8	3.1	26.98	0.06	0.02138	4	0.974	1
2463	2883.9	3.1	27.04	0.07	0.02142	3	0.975	1
2462	2887.0	3.1	27.11	0.07	0.02145	3	0.976	2
2461	2890.1	3.1	27.18	0.06	0.02148	4	0.978	1
2460	2893.2	3.1	27.24	0.07	0.02152	3	0.979	1
2459	2896.3	3.0	27.31	0.07	0.02155	3	0.980	1
2458	2899.3	3.1	27.38	0.06	0.02158	4	0.981	1
2457	2902.4	3.1	27.44	0.07	0.02162	3	0.982	2
2456	2905.5	3.1	27.51	0.07	0.02165	3	0.984	1
2455	2908.6	3.1	27.58	0.06	0.02168	3	0.985	1
2454	2911.7	3.1	27.64	0.07	0.02171	4	0.986	1
2453	2914.8	3.1	27.71	0.07	0.02175	3	0.987	2
2452	2917.9	3.1	27.78	0.06	0.02178	3	0.989	1
2451	2921.0	3.1	27.84	0.07	0.02181	4	0.990	1

TABLE I—CONTINUED.

V	S_v	*Diff.*	A_v	*Diff.*	I_v	*Diff.*	T_v	*Diff.*
2450	2924.1	3.1	27.91	0.06	0.02185	3	0.991	1
2449	2927.2	3.1	27.97	0.07	0.02188	3	0.992	1
2448	2930.3	3.1	28.04	0.07	0.02191	4	0.993	2
2447	2933.4	3.1	28.11	0.07	0.02195	3	0.995	1
2446	2936.5	3.1	28.18	0.07	0.02198	3	0.996	1
2445	2939.6	3.1	28.25	0.07	0.02201	4	0.997	2
2444	2942.7	3.1	28.32	0.07	0.02205	3	0.999	1
2443	2945.8	3.1	28.39	0.07	0.02208	4	1.000	1
2442	2948.9	3.2	28.46	0.07	0.02212	3	1.001	2
2441	2952.1	3.1	28.53	0.07	0.02215	4	1.003	1
2440	2955.2	3.1	28.60	0.07	0.02219	3	1.004	1
2439	2958.3	3.1	28.67	0.07	0.02222	3	1.005	1
2438	2961.4	3.2	28.74	0.07	0.02225	4	1.006	2
2437	2964.6	3.1	28.81	0.07	0.02229	3	1.008	1
2436	2967.7	3.1	28.88	0.07	0.02232	4	1.009	1
2435	2970.8	3.1	28.95	0.07	0.02236	3	1.010	1
2434	2973.9	3.1	29.02	0.07	0.02239	3	1.011	2
2433	2977.0	3.2	29.09	0.07	0.02242	4	1.013	1
2432	2980.2	3.1	29.16	0.07	0.02246	3	1.014	1
2431	2983.3	3.1	29.23	0.07	0.02249	4	1.015	1
2430	2986.4	3.1	29.30	0.07	0.02253	3	1.016	1
2429	2989.5	3.2	29.37	0.07	0.02256	3	1.017	2
2428	2992.7	3.1	29.44	0.07	0.02259	4	1.019	1
2427	2995.8	3.1	29.51	0.07	0.02263	3	1.020	1
2426	2998.9	3.2	29.58	0.08	0.02266	4	1.021	2
2425	3002.1	3.1	29.66	0.07	0.02270	3	1.023	1
2424	3005.2	3.2	29.73	0.07	0.02273	3	1.024	1
2423	3008.4	3.1	29.80	0.07	0.02276	4	1.025	2
2422	3011.5	3.1	29.87	0.07	0.02280	3	1.027	1
2421	3014.6	3.2	29.94	0.07	0.02283	4	1.028	1
2420	3017.8	3.1	30.01	0.07	0.02287	3	1.029	1
2419	3020.9	3.1	30.08	0.08	0.02290	4	1.030	2
2418	3024.0	3.2	30.16	0.07	0.02294	3	1.032	1
2417	3027.2	3.1	30.23	0.07	0.02297	4	1.033	1
2416	3030.3	3.2	30.30	0.07	0.02301	3	1.034	2
2415	3033.5	3.1	30.37	0.07	0.02304	4	1.036	1
2414	3036.6	3.1	30.44	0.08	0.02308	3	1.037	1
2413	3039.7	3.2	30.52	0.07	0.02311	4	1.038	2
2412	3042.9	3.1	30.59	0.07	0.02315	3	1.040	1
2411	3046.0	3.2	30.66	0.07	0.02318	4	1.041	1

TABLE I—CONTINUED.

V	S_v	*Diff.*	A_v	*Diff.*	I_v	*Diff.*	T_v	*Diff.*
2410	3049.2	3.1	30.73	0.07	0.02322	3	1.042	1
2409	3052.3	3.2	30.80	0.07	0.02325	4	1.043	1
2408	3055.5	3.2	30.87	0.08	0.02329	3	1.044	2
2407	3058.7	3.1	30.95	0.07	0.02332	4	1.046	1
2406	3061.8	3.1	31.02	0.07	0.02336	3	1.047	1
2405	3064.9	3.2	31.09	0.08	0.02339	4	1.048	2
2404	3068.1	3.2	31.17	0.07	0.02343	3	1.050	1
2403	3071.3	3.1	31.24	0.07	0.02346	4	1.051	1
2402	3074.4	3.2	31.31	0.08	0.02350	3	1.052	2
2401	3077.6	3.2	31.39	0.07	0.02353	4	1.054	1
2400	3080.8	3.2	31.46	0.07	0.02357	3	1.055	1
2399	3084.0	3.2	31.53	0.08	0.02360	4	1.056	1
2398	3087.2	3.2	31.61	0.07	0.02364	4	1.057	2
2397	3090.4	3.1	31.68	0.08	0.02368	3	1.059	1
2396	3093.5	3.2	31.76	0.07	0.02371	4	1.060	1
2395	3096.7	3.1	31.83	0.08	0.02375	4	1.061	2
2394	3099.8	3.2	31.91	0.07	0.02379	3	1.063	1
2393	3103.0	3.2	31.98	0.08	0.02382	4	1.064	1
2392	3106.2	3.2	32.06	0.07	0.02386	4	1.065	2
2391	3109.4	3.1	32.13	0.08	0.02390	3	1.067	1
2390	3112.5	3.1	32.21	0.07	0.02393	4	1.068	1
2389	3115.6	3.2	32.28	0.08	0.02397	4	1.069	1
2388	3118.8	3.2	32.36	0.08	0.02401	3	1.070	2
2387	3122.0	3.2	32.44	0.07	0.02404	4	1.072	1
2386	3125.2	3.2	32.51	0.08	0.02408	4	1.073	1
2385	3128.4	3.2	32.59	0.08	0.02412	3	1.074	2
2384	3131.6	3.2	32.67	0.07	0.02415	4	1.076	1
2383	3134.8	3.2	32.74	0.08	0.02419	4	1.077	1
2382	3138.0	3.2	32.82	0.08	0.02423	3	1.078	2
2381	3141.2	3.2	32.90	0.08	0.02426	3	1.080	1
2380	3144.4	3.2	32.98	0.07	0.02429	4	1.081	1
2379	3147.6	3.2	33.05	0.08	0.02433	4	1.082	1
2378	3150.8	3.2	33.13	0.08	0.02437	3	1.083	2
2377	3154.0	3.2	33.21	0.08	0.02440	4	1.085	1
2376	3157.2	3.2	33.29	0.08	0.02444	3	1.086	2
2375	3160.4	3.2	33.37	0.08	0.02447	4	1.088	1
2374	3163.6	3.2	33.45	0.08	0.02451	4	1.089	2
2373	3166.8	3.2	33.53	0.08	0.02455	3	1.091	1
2372	3170.0	3.2	33.61	0.08	0.02458	4	1.092	2
2371	3173.2	3.2	33.69	0.08	0.02462	3	1.094	1

TABLE I—Continued.

V	S_v	*Diff.*	A_v	*Diff.*	I_v	*Diff.*	T_v	*Diff.*
2370	3176.4	3.2	33.77	0.08	0.02465	4	1.095	1
2369	3179.6	3.2	33.85	0.08	0.02469	3	1.096	2
2368	3182.8	3.2	33.93	0.08	0.02472	4	1.098	1
2367	3186.0	3.2	34.01	0.08	0.02476	4	1.099	1
2366	3189.2	3.2	34.09	0.08	0.02480	3	1.100	2
2365	3192.4	3.2	34.17	0.08	0.02483	4	1.102	1
2364	3195.6	3.2	34.25	0.08	0.02487	4	1.103	2
2363	3198.8	3.3	34.33	0.08	0.02491	3	1.105	1
2362	3202.1	3.2	34.41	0.08	0.02494	4	1.106	2
2361	3205.3	3.2	34.49	0.08	0.02498	4	1.108	1
2360	3208.5	3.2	34.57	0.08	0.02502	4	1.109	1
2359	3211.7	3.2	34.65	0.08	0.02506	3	1.110	2
2358	3214.9	3.3	34.73	0.08	0.02509	4	1.112	1
2357	3218.2	3.2	34.81	0.08	0.02513	4	1.113	1
2356	3221.4	3.2	34.89	0.08	0.02517	3	1.114	2
2355	3224.6	3.2	34.97	0.09	0.02520	4	1.116	1
2354	3227.8	3.2	35.06	0.08	0.02524	4	1.117	2
2353	3231.0	3.2	35.14	0.08	0.02528	3	1.119	1
2352	3234.2	3.3	35.22	0.08	0.02531	4	1.120	2
2351	3237.5	3.2	35.30	0.08	0.02535	4	1.122	1
2350	3240.7	3.2	35.38	0.08	0.02539	4	1.123	1
2349	3243.9	3.3	35.46	0.08	0.02543	3	1.124	2
2348	3247.2	3.2	35.54	0.09	0.02546	4	1.126	1
2347	3250.4	3.2	35.63	0.08	0.02550	4	1.127	2
2346	3253.6	3.3	35.71	0.08	0.02554	4	1.129	1
2345	3256.9	3.2	35.79	0.09	0.02558	3	1.130	2
2344	3260.1	3.2	35.88	0.08	0.02561	4	1.132	1
2343	3263.3	3.3	35.96	0.08	0.02565	4	1.133	1
2342	3266.6	3.2	36.04	0.09	0.02569	4	1.134	2
2341	3269.8	3.3	36.13	0.08	0.02573	4	1.136	1
2340	3273.1	3.2	36.21	0.08	0.02577	3	1.137	1
2339	3276.3	3.3	36.29	0.09	0.02580	4	1.138	2
2338	3279.6	3.2	36.38	0.08	0.02584	4	1.140	1
2337	3282.8	3.3	36.46	0.08	0.02588	4	1.141	1
2336	3286.1	3.2	36.54	0.09	0.02592	3	1.142	2
2335	3289.3	3.3	36.63	0.08	0.02595	4	1.144	1
2334	3292.6	3.3	36.71	0.09	0.02599	4	1.145	2
2333	3295.9	3.2	36.80	0.08	0.02603	4	1.147	1
2332	3299.1	3.3	36.88	0.09	0.02607	4	1.148	2
2331	3302.4	3.3	36.97	0.08	0.02611	4	1.150	1

TABLE I—CONTINUED.

V	S_v	*Diff.*	A_v	*Diff.*	I_v	*Diff.*	T_v	*Diff.*
2330	3305.7	3.2	37.05	0.09	0.02615	3	1.151	1
2329	3308.9	3.3	37.14	0.08	0.02618	4	1.152	2
2328	3312.2	3.3	37.22	0.09	0.02622	4	1.154	1
2327	3315.5	3.3	37.31	0.08	0.02626	4	1.155	2
2326	3318.8	3.3	37.39	0.09	0.02630	4	1.157	1
2325	3322.1	3.3	37.48	0.08	0.02634	4	1.158	1
2324	3325.4	3.2	37.56	0.09	0.02638	4	1.159	2
2323	3328.6	3.3	37.65	0.09	0.02642	4	1.161	1
2322	3331.9	3.3	37.74	0.08	0.02646	4	1.162	2
2321	3335.2	3.3	37.82	0.09	0.02650	4	1.164	1
2320	3338.5	3.2	37.91	0.09	0.02654	4	1.165	1
2319	3341.7	3.3	38.00	0.08	0.02658	4	1.166	2
2318	3345.0	3.3	38.08	0.09	0.02662	4	1.168	1
2317	3348.3	3.3	38.17	0.09	0.02666	4	1.169	2
2316	3351.6	3.2	38.26	0.08	0.02670	4	1.171	1
2315	3354.8	3.3	38.34	0.09	0.02674	4	1.172	1
2314	3358.1	3.3	38.43	0.09	0.02678	4	1.173	2
2313	3361.4	3.3	38.52	0.09	0.02682	4	1.175	1
2312	3364.7	3.3	38.61	0.09	0.02686	4	1.176	2
2311	3368.0	3.3	38.70	0.09	0.02690	4	1.178	1
2310	3371.3	3.2	38.79	0.09	0.02694	4	1.179	1
2309	3374.5	3.3	38.88	0.09	0.02698	4	1.180	2
2308	3377.8	3.3	38.97	0.09	0.02702	4	1.182	1
2307	3381.1	3.3	39.06	0.09	0.02706	4	1.183	1
2306	3384.4	3.3	39.15	0.09	0.02710	4	1.184	2
2305	3387.7	3.3	39.24	0.09	0.02714	4	1.186	1
2304	2391.0	3.3	39.33	0.09	0.02718	4	1.187	2
2303	3394.3	3.3	39.42	0.09	0.02722	4	1.189	1
2302	3397.6	3.3	39.51	0.09	0.02726	4	1.190	2
2301	3400.9	3.3	39.60	0.09	0.02730	4	1.192	1
2300	3404.2	3.3	39.69	0.09	0.02734	4	1.193	1
2299	3407.5	3.3	39.78	0.09	0.02738	4	1.194	2
2298	3410.8	3.3	39.87	0.09	0.02742	4	1.196	1
2297	3414.1	.33	39.96	0.09	0.02746	4	1.197	1
2296	3417.4	3.3	40.05	0.10	0.02750	5	1.198	2
2295	3420.7	3.3	40.15	0.09	0.02755	4	1.200	1
2294	3424.0	3.3	40.24	0.09	0.02759	4	1.201	2
2293	3427.3	3.4	40.33	0.09	0.02763	4	1.203	1
2292	3430.7	3.3	40.42	0.09	0.02767	4	1.204	2
2291	3434.0	3.3	40.51	0.09	0.02771	4	1.206	1

TABLE I—Continued.

V	S_v	*Diff.*	A_v	*Diff.*	I_v	*Diff.*	T_v	*Diff.*
2290	3437.3	3.3	40.60	0.09	0.02775	4	1.207	1
2289	3440.6	3.3	40.69	0.09	0.02779	4	1.208	2
2288	3443.9	3.4	40.78	0.10	0.02783	4	1.210	1
2287	3447.3	3.3	40.88	0.09	0.02787	4	1.211	1
2286	3450.6	3.4	40.97	0.09	0.02791	5	1.212	2
2285	3454.0	3.3	41.06	0.10	0.02796	4	1.214	1
2284	3457.3	3.3	41.16	0.09	0.02800	4	1.215	2
2283	3460.6	3.4	41.25	0.09	0.02804	4	1.217	1
2282	3464.0	3.3	41.34	0.10	0.02808	4	1.218	2
2281	3467.3	3.3	41.44	0.09	0.02812	4	1.220	1
2280	3470.6	3.3	41.53	0.09	0.02816	4	1.221	1
2279	3473.9	3.4	41.62	0.10	0.02820	4	1.222	2
2278	3477.3	3.3	41.72	0.09	0.02824	4	1.224	1
2277	3480.6	3.4	41.81	0.10	2.02828	4	1.225	2
2276	3484.0	3.3	41.91	0.09	0.02832	5	1.227	1
2275	3487.3	3.4	42.00	0.10	0.02837	4	1.228	2
2274	3490.7	3.3	42.10	0.09	0.02841	4	1.230	1
2273	3494.0	3.4	42.19	0.10	0.02845	4	1.231	2
2272	3497.4	3.3	42.29	0.09	0.02849	4	1.233	1
2271	3500.7	3.4	42.38	0.10	0.02853	4	1.234	2
2270	3504.1	3.3	42.48	0.10	0.02857	4	1.236	1
2269	3507.4	3.4	42.58	0.09	0.02861	4	1.237	2
2268	3510.8	3.3	42.67	0.10	0.02865	5	1.239	1
2267	3514.1	3.4	42.77	0.10	0.02870	4	1.240	2
2266	3517.5	3.3	42.87	0.09	0.02874	4	1.242	1
2265	3520.8	3.4	42.96	0.10	0.02878	4	1.243	2
2264	3524.2	3.3	43.06	0.10	0.02882	5	1.245	1
2263	3527.5	3.4	43.16	0.09	0.02887	4	1.246	2
2262	3530.9	3.3	43.25	0.10	0.02891	4	1.248	1
2261	3534.2	3.4	43.35	0.10	0.02895	4	1.249	2
2260	3537.6	3.3	43.45	0.10	0.02899	4	1.251	1
2259	3540.9	3.4	43.55	0.10	0.02903	5	1.252	2
2258	3544.3	3.3	43.65	0.09	0.02908	4	1.254	1
2257	3547.6	3.4	43.74	0.10	0.02912	4	1.255	2
2256	3551.0	3.3	43.84	0.10	0.02916	4	1.257	1
2255	3554.3	3.4	43.94	0.09	0.02920	4	1.258	2
2254	3557.7	3.4	44.03	0.10	0.02924	5	1.260	1
2253	3561.1	3.4	44.13	0.10	0.02929	4	1.261	2
2252	3564.5	3.3	44.23	0.10	0.02933	4	1.263	1
2251	3567.8	3.4	44.33	0.10	0.02937	4	1.264	2

TABLE I—CONTINUED.

V	S_v	*Diff.*	A_v	*Diff.*	I_v	*Diff.*	T_v	*Diff.*
2250	3571.2	3.4	44.43	0.10	0.02941	4	1.266	1
2249	3574.6	3.3	44.53	0.10	0.02945	4	1.267	2
2248	3577.9	3.4	44.63	0.10	0.02949	5	1.269	1
2247	3581.3	3.3	44.73	0.10	0.02954	4	1.270	2
2246	3584.6	3.4	44.83	0.10	0.02958	4	1.272	1
2245	3588.0	3.4	44.93	0.10	0.02962	5	1.273	2
2244	3591.4	3.4	45.03	0.10	0.02967	4	1.275	1
2243	3594.8	3.4	45.13	0.10	0.02971	4	1.276	2
2242	3598.2	3.4	45.23	0.10	0.02975	5	1.278	1
2241	3601.6	3.4	45.33	0.10	0.02980	4	1.279	2
2240	3605.0	3.4	45.43	0.10	0.02984	4	1.281	1
2239	3608.4	3.4	45.53	0.10	0.02988	5	1.282	2
2238	3611.8	3.4	45.63	0.10	0.02993	4	1.284	1
2237	3615.2	3.4	45.73	0.11	0.02997	5	1.285	2
2236	3618.6	3.4	45.84	0.10	0.03002	4	1.287	1
2235	3622.0	3.4	45.94	0.10	0.03006	4	1.288	2
2234	3625.4	3.4	46.04	0.10	0.03010	5	1.290	1
2233	3628.8	3.4	46.14	0.11	0.03015	4	1.291	2
2232	3632.2	3.4	46.25	0.10	0.03019	5	1.293	1
2231	3635.6	3.4	46.35	0.10	0.03024	4	1.294	2
2230	3639.0	3.4	46.45	0.10	0.03028	4	1.296	1
2229	3642.4	3.4	46.55	0.10	0.03032	5	1.297	2
2228	3645.8	3.4	46.65	0.11	0.03037	4	1.299	1
2227	3649.2	3.4	46.76	0.10	0.03041	5	1.300	2
2226	3652.6	3.5	46.86	0.10	0.03046	4	1.302	1
2225	3656.1	3.4	46.96	0.11	0.03050	4	1.303	2
2224	3659.5	3.4	47.07	0.10	0.03054	5	1.305	1
2223	3662.9	3.4	47.17	0.11	0.03059	4	1.306	2
2222	3666.3	3.5	47.28	0.10	0.03063	5	1.308	1
2221	3669.8	3.4	47.38	0.11	0.03068	4	1.309	2
2220	3673.2	3.4	47.49	0.10	0.03072	4	1.311	1
2219	3676.6	3.4	47.59	0.11	0.03076	5	1.312	2
2218	3680.0	3.5	47.70	0.11	0.03081	4	1.314	1
2217	3683.5	3.4	47.81	0.10	0.03085	5	1.315	2
2216	3686.9	3.4	47.91	0.11	0.03090	4	1.317	1
2215	3690.3	3.5	48.02	0.11	0.03094	5	1.318	2
2214	3693.8	3.4	48.13	0.10	0.03099	4	1.320	2
2213	3697.2	3.4	48.23	0.11	0.03103	5	1.322	1
2212	3700.6	3.5	48.34	0.10	0.03108	4	1.323	2
2211	3704.1	3.4	48.44	0.11	0.03112	5	1.325	2

TABLE I—Continued.

V	S_v	*Diff.*	A_v	*Diff.*	I_v	*Diff.*	T_v	*Diff.*
2210	3707.5	3.4	48.55	0.10	0.03117	4	1.327	1
2209	3710.9	3.5	48.65	0.11	0.03121	5	1.328	2
2208	2714.4	3.4	48.76	0.11	0.03126	5	1.330	2
2207	3717.8	3.5	48.87	0.10	0.03131	4	1.332	1
2206	3721.3	3.4	48.97	0.11	0.03135	5	1.333	2
2205	3724.7	3.5	49.08	0.11	0.03140	4	1.335	2
2204	3728.2	3.4	49.19	0.11	0.03144	5	1.337	1
2203	3731.6	3.5	49.30	0.11	0.03149	5	1.338	2
2202	3735.1	3.4	49.41	0.11	0.03154	4	1.340	1
2201	3738.5	3.5	49.52	0.11	0.03158	5	1.341	2
2200	3742.0	3.5	49.63	0.11	0.03163	5	1.343	1
2199	3745.5	3.4	49.74	0.11	0.03168	4	1.344	2
2198	3748.9	3.5	49.85	0.11	0.03172	5	1.346	2
2197	3752.4	3.4	49.96	0.11	0.03177	5	1.348	1
2196	3755.8	3.5	50.07	0.11	0.03182	4	1.349	2
2195	3759.3	3.5	50.18	0.11	0.03186	5	1.351	2
2194	3762.8	3.5	50.29	0.11	0.03191	5	1.353	1
2193	3766.3	3.4	50.40	0.11	0.03196	4	1.354	2
2192	3769.7	3.5	50.51	0.11	0.03200	5	1.356	1
2191	3773.2	3.5	50.62	0.11	0.03205	4	1.357	2
2190	3776.7	3.4	50.73	0.11	0.03209	5	1.359	1
2189	3780.1	3.5	50.84	0.11	0.03214	5	1.360	2
2188	3783.6	3.5	50.95	0.11	0.03219	4	1.362	1
2187	3787.1	3.4	51.06	0.11	0.03223	5	1.363	2
2186	3790.5	3.5	51.17	0.12	0.03228	5	1.365	2
2185	3794.0	3.5	51.29	0.11	0.03233	4	1.367	1
2184	3797.5	3.4	51.40	0.11	0.03237	5	1.368	2
2183	3800.9	3.5	51.51	0.11	0.03242	5	1.370	2
2182	3804.4	3.5	51.62	0.12	0.03247	4	1.372	1
2181	2807.9	3.5	51.74	0.11	0.03251	5	1.373	2
2180	3811.4	3.5	51.85	0.11	0.03256	5	1.375	1
2179	3814.9	3.5	51.96	0.12	0.03261	5	1.376	2
2178	3818.4	3.5	52.08	0.11	0.03266	4	1.378	1
2177	3821.9	3.5	52.19	0.12	0.03270	5	1.379	2
2176	3825.4	3.5	52.31	0.11	0.03275	5	1.381	2
2175	3828.9	3.5	52.42	0.12	0.03280	5	1.383	1
2174	3832.4	3.5	52.54	0.11	0.03285	4	1.384	2
2173	3835.9	3.5	52.65	0.12	0.03289	5	1.386	2
2172	3839.4	3.5	52.77	0.11	0.03294	5	1.388	1
2171	3842.9	3.5	52.88	0.12	0.03299	5	1.389	2

TABLE I—CONTINUED.

V	S_v	*Diff.*	A_v	*Diff.*	I_v	*Diff.*	T_v	*Diff.*
2170	3846.4	3.5	53.00	0.11	0.03304	4	1.391	2
2169	3849.9	3.5	53.11	0.12	0.03308	5	1.393	1
2168	3853.4	3.5	53.23	0.12	0.03313	4	1.394	2
2167	3856.9	3.5	53.35	0.11	0.03317	5	1.396	2
2166	3860.4	3.5	53.46	0.12	0.03322	5	1.398	1
2165	3663.9	3.5	53.58	0.12	0.03327	5	1.399	2
2164	3867.4	3.5	53.70	0.11	0.03332	5	1.401	2
2163	3870.9	3.5	53.81	0.12	0.03337	5	1.403	1
2162	3874.4	3.5	53.93	0.12	0.03342	5	1.404	2
2161	3877.9	3.5	54.05	0.12	0.03347	5	1.406	1
2160	3881.4	3.5	54.17	0.12	0.03352	5	1.407	2
2159	3884.9	3.5	54.29	0.12	0.03357	5	1.409	1
2158	3888.4	3.5	54.41	0.12	0.03362	4	1.410	2
2157	3891.9	3.6	54.53	0.12	0.03366	5	1.412	2
2156	3895.5	3.5	54.65	0.12	0.03371	5	1.414	1
2155	3899.0	3.5	54.77	0.12	0.03376	5	1.415	2
2154	3902.5	3.6	54.89	0.12	0.03381	5	1.417	2
2153	3906.1	3.5	55.01	0.12	0.03386	5	1.419	1
2152	3909.6	3.5	55.13	0.12	0.03391	5	1.420	2
2151	3913.1	3.6	55.25	0.12	0.03396	5	1.422	1
2150	3916.7	3.5	55.37	0.12	0.03401	4	2.423	2
2149	3920.2	3.6	55.49	0.12	0.03405	5	1.425	2
2148	3923.8	3.5	55.61	0.12	0.03410	5	1.427	1
2147	2927.3	3.5	55.73	0.12	0.03415	5	1.428	2
2146	3930.8	3.6	55.85	0.13	0.03420	5	1.430	2
2145	3934.4	3.5	55.98	0.12	0.03425	5	1.432	1
2144	3937.9	3.6	56.10	0.12	0.03430	5	1.433	2
2143	3941.5	3.5	56.22	0.12	0.03435	5	1.435	2
2142	3945.0	3.6	56.34	0.13	0.03440	5	1.437	1
2141	3948.6	3.5	56.47	0.12	0.03445	5	1.438	2
2140	3952.1	3.5	56.59	0.12	0.03450	5	1.440	2
2139	3955.6	3.6	56.71	0.13	0.03455	5	1.442	2
2138	3959.2	3.5	56.84	0.12	0.03460	5	1.444	1
2137	3962.7	3.6	56.96	0.12	0.03465	5	1.445	2
2136	3966.3	3.5	57.08	0.13	0.03470	5	1.447	2
2135	3969.8	3.6	57.21	0.12	0.03475	6	1.449	1
2134	3973.4	3.5	57.33	0.13	0.03481	5	1.450	2
2133	2976.9	3.6	57.46	0.12	0.03486	5	1.452	2
2132	3980.5	3.5	57.58	0.13	0.03491	5	1.454	1
2131	3984.0	3.6	57.71	0.12	0.03496	5	1.455	2

TABLE I—Continued.

V	S_v	*Diff.*	A_v	*Diff.*	I_v	*Diff.*	T_v	*Diff.*
2130	3987.6	3.5	57.83	0.13	0.03501	5	1.457	2
2129	3991.1	3.6	57.96	0.12	0.03506	6	1.459	2
2128	3994.7	3.5	58.08	0.13	0.03512	5	1.461	1
2127	3998.2	3.6	58.21	0.12	0.03517	5	1.462	2
2126	4001.8	3.6	58.33	0.13	0.03522	5	1.464	2
2125	4005.4	3.6	58.46	0.12	0.03527	5	1.466	1
2124	4009.0	3.6	58.58	0.13	0.03532	5	1.467	2
2123	4012.6	3.6	58.71	0.13	0.03537	5	1.469	2
2122	4016.2	3.6	58.84	3.12	0.03542	5	1.471	1
2121	4019.8	3.6	58.96	0.13	0.03547	5	1.472	2
2120	4023.4	3.5	59.09	0.13	0.03552	5	1.474	2
2119	4026.9	3.6	59.22	0.13	0.03557	6	1.476	1
2118	4030.5	3.6	59.35	0.12	0.03563	5	1.477	2
2117	4034.1	3.6	59.47	0.13	0.03568	5	1.479	2
2116	4037.7	3.6	59.60	0.13	0.03573	5	1.481	1
2115	4041.3	3.6	59.73	0.12	0.03578	5	1.482	2
2114	4044.9	3.6	59.85	0.13	0.03583	5	1.484	2
2113	4048.5	3.6	59.98	0.13	0.03588	5	1.486	1
2112	4052.1	3.6	60.11	0.13	0.03593	5	1.487	2
2111	4055.7	3.6	60.24	0.13	0.03598	5	1.489	2
2110	4059.3	3.6	60.37	0.13	0.03603	5	1.491	2
2109	4062.9	3.6	60.50	0.13	0.03608	5	1.493	1
2108	4066.5	3.6	60.63	0.13	0.03613	6	1.494	2
2107	4070.1	3.7	60.76	0.13	0.03619	5	1.496	2
2106	4073.8	3.6	60.89	0.13	0.03624	5	1.498	1
2105	4077.4	3.6	61.02	0.13	0.03629	6	1.499	2
2104	4081.0	3.6	61.15	0.13	0.03635	5	1.501	2
2103	4084.6	3.7	61.28	0.13	0.03640	5	1.503	1
2102	4088.3	3.6	61.41	0.13	0.03645	6	1.504	2
2101	4091.9	3.6	61.54	0.13	0.03651	5	1.506	2
2100	4095.5	3.6	61.67	0.13	0.03656	5	1.508	1
2099	4099.1	3.6	61.80	0.13	0.03661	5	1.509	2
2098	4102.7	3.6	61.93	0.14	0.03666	6	1.511	2
2097	4106.3	3.6	62.07	0.13	0.03672	5	1.513	1
2096	4109.9	3.7	62.20	0.13	0.03677	5	1.514	2
2095	4113.6	3.6	62.33	0.14	0.03682	6	1.516	2
2094	4117.2	3.7	62.47	0.13	0.03688	5	1.518	2
2093	4120.9	3.6	62.60	0.13	0.03693	5	1.520	1
2092	4124.5	3.6	62.73	0.14	0.03698	6	1.521	2
2091	4128.1	3.7	62.87	0.13	0.03704	5	1.523	2

TABLE I—Continued.

V	*S*v	*Diff.*	*A*v	*Diff.*	*I*v	*Diff.*	*T*v	*Diff.*
2090	4131.8	3.6	63.00	0.14	0.03709	5	1.525	2
2089	4135.4	3.6	63.14	0.13	0.03714	5	1.527	2
2088	4139.0	3.7	63.27	0.14	0.03719	6	1.529	1
2087	4142.7	3.6	63.41		0.03725	5	1.530	2
2086	4146.3	3.7	63.54	0.14	0.03730	6	1.532	2
2085	4150.0	3.6	63.68	0.13	0.03736	5	1.534	2
2084	4153.6	3.7	63.81	0.14	0.03741	5	1.536	1
2083	4157.3	3.6	63.95	0.13	0.03746	6	1.537	2
2082	4160.9	3.6	64.08	0.14	0.03752	5	1.539	2
2081	4164.5	3.7	64.22	0.14	0.03757	6	1.541	2
2080	4168.2	3.6	64.36	0.13	0.03763	5	1.543	2
2079	4171.8	3.7	64.49	0.14	0.03768	5	1.545	1
2078	4175.5	3.6	94.63	0.14	0.03773	6	1.546	2
2077	4179.1	3.7	64.77	0.14	0.03779	5	1.548	2
2076	4182.8	3.6	64.91	0.14	0.03784	6	1.550	1
2075	4186.4	3.7	65.05	0.14	0.03790	5	1.551	2
2074	4190.1	3.6	65.19	0.14	0.03795	5	1.553	2
2073	4193.7	3.7	65.33	0.14	0.03800	6	1.555	2
2072	4197.4	3.6	65.47	0.14	0.03806	5	1.557	1
2071	4201.0	3.7	65.61	0.14	0.03811	6	1.558	2
2070	4204.7	3.6	65.75	0.14	0.03817	5	1.560	2
2069	4208.3	3.7	65.89	0.14	0.03822	6	1.562	2
2068	4212.0	3.6	66.03	0.14	0.03828	6	1.564	2
2067	4215.6	3.7	66.17	0.15	0.03834	5	1.566	1
2066	4219.3	3.7	66.32	0.14	0.03839	6	1.567	2
2065	4223.0	3.7	66.46	0.14	0.03845	5	1.569	2
2064	4226.7	3.7	66.60	0.14	0.03850	6	1.571	2
2063	4230.4	3.7	66.74	0.15	0.03856	6	1.573	1
2062	4234.1	3.7	66.89	0.14	0.03862	5	1.574	2
2061	4237.8	3.7	67.03	0.14	0.03867	6	1.576	2
2060	4241.5	3.7	67.17	0.14	0.03873	6	1.578	2
2059	4245.2	3.7	67.31	0.15	0.03879	5	1.580	2
2058	4248.9	3.7	67.46	0.14	0.03884	6	1.582	1
2057	4252.6	3.7	67.60	0.15	0.03890	6	1.583	2
2056	4256.3	3.7	67.75	0.14	0.03896	5	1.585	2
2055	4260.0	3.7	67.89	0.15	0.03901	6	1.587	2
2054	4263.7	3.7	68.04	0.14	0.03907	6	1.589	2
2053	4267.4	3.7	68.18	0.15	0.03913	5	1.591	1
2052	4271.1	3.7	68.33	0.14	0.03918	6	1.592	2
2051	4274.8	3.8	68.47	0.15	0.03924	5	1.594	2

TABLE I—CONTINUED.

V	S_v	*Diff.*	A_v	*Diff.*	I_v	*Diff.*	T_v	*Diff.*
2050	4278.6	3.7	68.62	0.14	0.03929	6	1.596	2
2049	4282.3	3.7	68.76	0.15	0.03935	6	1.598	2
2046	4286.0	3.7	68.91	0.15	0.03941	5	1.600	2
2047	4289.7	3.7	69.06	0.14	0.03946	6	1.602	1
2046	4293.4	3.7	69.20	0.15	0.03952	6	1.603	2
2045	4297.1	3.8	69.35	0.15	0.03958	5	1.605	2
2044	4300.9	3.7	69.50	0.14	0.03963	6	1.607	2
2043	4304.6	3.7	69.64	0.15	0.03969	6	1.609	1
2042	4308.3	3.7	69.79	0.15	0.03975	5	1.610	2
2041	4312.0	3.8	69.94	0.15	0.03980	6	1.612	2
2040	4315.8	3.7	70.09	0.15	0.03986	6	1.614	2
2039	4319.5	3.8	70.24	0.15	0.03992	6	1.616	2
2038	3323.3	3.7	70.39	0.15	0.03998	5	1.618	1
2037	4327.0	3.7	70.54	0.15	0.04003	6	1.619	2
2036	4330.7	3.7	70.69	0.15	0.04009	6	1.621	2
2035	4334.4	3.8	70.84	0.15	0.04015	6	1.623	2
2034	4338.2	3.7	70.99	0.15	0.04021	5	1.625	2
2033	4341.9	3.7	71.14	0.15	0.04026	6	1.627	1
2032	4345.6	3.8	71.29	0.15	0.04032	6	1.628	2
2031	4349.4	3.7	71.44	0.15	0.04038	6	1.630	2
2030	4353.1	3.7	71.59	0.15	0.04044	6	1.632	2
2029	4356.8	3.8	71.74	0.15	0.04050	5	1.634	2
2028	4360.6	3.7	71.89	0.16	0.04055	6	1.636	2
2027	4364.3	3.8	72.05	0.15	0.04061	6	1.638	2
2026	4368.1	3.7	72.20	0.15	0.04067	6	1.640	2
2025	4371.8	3.8	72.35	0.15	0.04073	6	1.642	1
2024	4375.6	3.8	72.50	0.16	0.04079	6	1.643	2
2023	4379.4	3.7	72.66	0.15	0.04085	6	1.645	2
2022	4383.1	3.8	72.81	0.15	0.04091	6	1.647	2
2021	4386.9	3.8	72.96	0.16	0.04097	6	1.649	2
2020	4390.7	3.7	73.12	0.16	0.04103	6	1.651	2
2019	4394.4	3.8	73.28	0.15	0.04109	6	1.653	2
2018	4398.2	3.7	73.43	0.16	0.04115	6	1.655	2
2017	4401.9	3.8	73.59	0.15	0.04121	6	1.657	2
2016	4405.7	3.7	73.74	0.16	0.04127	6	1.659	1
2015	4409.4	3.8	73.90	0.15	0.04133	6	1.660	2
2014	4413.2	3.8	74.05	0.16	0.04139	6	1.662	2
2013	4417.0	3.8	74.21	0.15	0.04145	6	1.664	2
2012	4420.8	3.8	74.36	0.16	0.04151	6	1.666	2
2011	4424.6	3.8	74.52	0.16	0.04157	6	1.668	2

TABLE I—CONTINUED.

V	S_v	*Diff.*	A_v	*Diff.*	I_v	*Diff.*	T_v	*Diff.*
2010	4428.4	3.7	74.68	0.16	0.04163	6	1.670	2
2009	4432.1	3.8	74.84	0.16	0.04169	6	1.672	2
2008	4435.9	3.7	75.00	0.16	0.04175	6	1.674	2
2007	4439.6	3.8	75.16	0.15	0.04181	6	1.676	2
2006	4443.4	3.8	75.31	0.16	0.04187	6	1.678	1
2005	4447.2	3.8	75.47	0.16	0.04193	7	1.679	2
2004	4451.0	3.8	75.63	0.16	0.04200	6	1.681	2
2003	4454.8	3.8	75.79	0.15	0.04206	6	1.683	2
2002	4458.6	3.8	75.94	0.16	0.04212	6	1.685	2
2001	4462.4	3.8	76.10	0.16	0.04218	6	1.687	2
2000	4466.2	3.8	76.26	0.16	0.04224	6	1.689	2
1999	4470.0	3.8	76.42	0.16	0.04230	6	1.691	2
1998	4473.8	3.8	76.58	0.16	0.04236	6	1.693	2
1997	4477.6	3.8	76.74	0.16	0.04242	6	1.695	2
1996	4481.4	3.8	76.90	0.17	0.04248	7	1.697	1
1995	4485.2	3.8	77.07	0.16	0.04255	6	1.698	2
1994	4489.0	3.8	77.23	0.16	0.04261	6	1.700	2
1993	4492.8	3.8	77.39	0.16	0.04267	6	1.702	2
1992	4496.6	3.9	77.55	0.16	0.04273	6	1.704	2
1991	4500.5	3.8	77.71	0.16	0.04279	6	1.706	2
1990	4504.3	3.8	77.87	0.16	0.04285	6	1.708	2
1989	4508.1	3.8	78.03	0.17	0.04291	6	1.710	2
1988	4511.9	3.8	78.20	0.16	0.04297	7	1.712	2
1987	4515.7	3.9	78.36	0.17	0.04304	6	1.714	2
1986	4519.6	3.8	78.53	0.16	0.04310	6	1.716	1
1985	4523.4	3.8	78.69	0.17	0.04316	7	1.717	2
1984	4527.2	3.9	78.86	0.16	0.04323	6	1.719	2
1983	4531.1	3.8	79.02	0.17	0.04329	6	1.721	2
1982	4534.9	3.9	79.19	0.16	0.04335	7	1.723	2
1981	4538.8	3.8	79.35	0.17	0.04342	6	1.725	2
1980	4542.6	3.8	79.52	0.16	0.04348	6	1.727	2
1979	5446.4	3.9	79.68	0.17	0.04354	6	1.729	2
1978	4550.3	3.8	79.85	0.17	0.04360	7	1.731	2
1977	4554.1	3.9	80.02	0.17	0.04367	6	1.733	2
1976	4558.0	3.8	80.19	0.17	0.04373	6	1.735	2
1975	4561.8	3.9	80.36	0.17	0.04379	7	1.737	2
1974	4565.7	3.8	80.53	0.17	0.04386	6	1.739	2
1973	4569.5	3.9	80.70	0.17	0.04392	6	1.741	2
1972	4573.4	3.8	80.87	0.17	0.04398	7	1.743	2
1971	4577.2	3.9	81.04	0.17	0.04405	6	1.745	2

TABLE I—CONTINUED.

V	S_v	*Diff.*	A_v	*Diff.*	I_v	*Diff.*	T_v	*Diff.*
1970	4581.1	3.8	81.21	0.17	0.04411	6	1.747	2
1969	4584.9	3.9	81.38	0.17	0.04417	7	1.749	1
1968	4588.8	3.9	81.55	0.17	0.04424	6	1.750	2
1967	4592.7	3.8	81.72	0.18	0.04430	6	1.752	2
1966	4596.5	3.9	81.90	0.17	0.04436	7	1.754	2
1965	4600.4	3.9	82.07	0.17	0.04443	6	1.756	2
1964	4604.3	3.8	82.24	0.17	0.04449	7	1.758	2
1963	4608.1	3.9	82.41	0.18	0.04456	6	1.760	2
1962	4612.0	3.9	82.59	0.17	0.04462	6	1.762	2
1961	4615.9	3.9	82.76	0.17	0.04468	7	1.764	2
1960	4619.8	3.9	82.93	0.18	0.04475	6	1.766	2
1959	4623.7	3.8	83.11	0.17	0.04481	7	1.768	2
1958	4627.5	3.9	83.28	0.18	0.04488	7	1.770	2
1957	4631.4	3.9	83.46	0.17	0.04495	6	1.772	2
1956	4635.3	3.9	83.63	0.18	0.04501	7	1.774	2
1955	4639.2	3.9	83.81	0.17	0.04508	7	1.776	2
1954	4643.1	3.9	83.98	0.18	0.04515	6	1.778	2
1953	4647.0	3.9	84.16	0.17	0.04521	7	1.780	2
1932	4650.9	3.9	84.33	0.18	0.04528	6	1.782	2
1951	4654.8	3.9	84.51	0.18	0.04534	7	1.784	2
1950	4658.7	3.9	84.69	0.17	0.04541	7	1.786	2
1949	4662.6	3.9	84.86	0.18	0.04548	6	1.788	2
1948	4666.5	3.9	85.04	0.18	0.04554	7	1.790	2
1947	4670.4	3.9	85.22	0.18	0.04561	7	1.792	2
1946	4674.3	3.9	85.40	0.18	0.04568	6	1.794	2
1945	4678.2	3.9	85.58	0.18	0.04574	7	1.796	2
1944	4682.1	3.9	85.76	0.18	0.04581	7	1.798	2
1943	4686.0	3.9	85.94	0.18	0.04588	6	1.800	2
1942	4689.9	3.9	86.12	0.18	0.04594	7	1.802	2
1941	4693.8	3.9	86.30	0.18	0.04601	6	1.804	2
1940	4697.7	3.9	86.48	0.18	0.04607	7	1.806	2
1939	4701.6	3.9	86.66	0.18	0.04614	7	1.808	2
1938	4705.5	3.9	86.84	0.18	0.04621	6	1.810	2
1937	4709.4	3.9	87.02	0.19	0.04627	7	1.812	2
1936	4713.3	4.0	87.21	0.18	0.04634	7	1.814	2
1935	4717.3	3.9	87.39	0.18	0.04641	7	1.816	2
1934	4721.2	3.9	87.57	0.18	0.04648	6	1.818	2
1933	4725.1	3.9	87.75	0.19	0.04654	7	1.820	2
1932	4729.0	4.0	87.94	0.18	0.64661	7	1.822	2
1931	4733.0	3.9	88.12	0.18	0.04668	7	1.824	2

TABLE I—CONTINUED.

V	S_v	*Diff.*	A_v	*Diff.*	I_v	*Diff.*	T_v	*Diff.*
1930	4736.9	3.9	88.30	0.19	0.04675	7	1.826	2
1929	4740.8	4.0	81.49	0.18	0.04682	6	1.828	2
1928	4744.8	3.9	88.67	0.19	0.04688	7	1.830	2
1927	4748.7	4.0	88.86	0.18	0.04695	7	1.832	2
1926	4752.7	3.9	89.04	9.19	0.04702	7	1.334	3
1925	4756.6	4.0	89.23	0.18	0.04709	7	1.837	2
1924	4760.6	3.9	89.41	0.19	0.04716	6	1.839	2
1923	4764.5	4.0	89.60	0.18	0.04722	7	1.841	2
1922	4768.5	3.9	89.78	0.19	0.04729	7	1.843	2
1921	4772.4	4.0	89.97	0.19	0.04736	7	1.845	2
1920	4776.4	3.9	90.16	0.19	0.04743	7	1.847	2
1919	4780.3	4.0	90.35	0.19	0.04750	7	1.849	2
1918	4784.3	3.9	90.54	0.19	0.04757	7	1.851	2
1917	4788.2	4.0	90.73	0.19	0.04764	7	1.853	2
1916	4792.2	3.9	90.92	0.19	0.04771	7	1.855	3
1915	4796.1	4.0	91.11	0.19	0.04678	7	1.858	2
1914	4800.1	4.0	91.30	0.19	0.04785	7	1.860	2
1913	4804.1	3.9	91.49	0.19	0.04792	7	1.862	2
1912	4808.0	4.0	91.68	0.19	0.04799	7	1.864	2
1911	4812.0	4.0	91.87	0.19	0.04806	7	1.866	2
1910	4816.0	3.9	92.06	0.19	0.04813	7	1.868	2
1909	4819.9	4.0	92.25	0.19	0.04820	7	1.870	2
1908	4823.9	3.9	92.44	0.20	0.04827	7	1.872	3
1907	4827.8	4.0	92.64	0.19	0.04834	7	1.875	2
1906	4831.8	4.0	92.83	0.19	0.04841	7	1.877	2
1905	4835.8	4.0	93.02	0.20	0.04848	8	1.879	2
1904	4839.8	4.0	93.22	0.19	0.04856	7	1.881	2
1903	4843.8	4.0	93.41	0.19	0.04863	7	1.883	2
1902	4847.8	4.0	93.60	0.20	0.04870	7	1.885	2
1901	4851.8	4.0	93.80	0.19	0.04877	7	1.887	2
1900	4855.8	4.0	93.99	0.20	0.04884	7	1.889	2
1899	4859.8	4.0	94.19	0.20	0.04891	7	1.891	2
1898	4863.8	4.0	94.39	0.19	0.04898	7	1.893	3
1897	4867.8	4.1	94 58	0.20	0.04905	7	1.896	2
1896	4871.9	4.0	94.78	0.20	0.04912	7	1.898	2
1895	4875.9	4.0	94.98	0.19	0.04919	8	1.900	2
1894	4879.9	4.0	95.17	0.20	0.04927	7	1.902	2
1893	4883.9	4.0	95.37	0.20	0.04934	7	1.904	2
1892	4887.9	4.1	95.57	0.19	0.04941	7	1.906	2
1891	4892.0	4.0	95.76	0.20	0.04948	7	1.908	2

TABLE I—CONTINUED.

V	S_v	*Diff.*	A_v	*Diff.*	I_v	*Diff.*	T_v	*Diff.*
1890	4896.0	4.0	95.96	0.20	0.04955	7	1.910	2
1889	4900.0	4.0	96.16	0.20	0.04962	8	1.912	2
1888	4904.0	4.1	96.36	0.20	0.04970	7	1.914	2
1887	4908.1	4.0	96.56	0.20	0.04977	7	1.916	3
1886	4912.1	4.1	96.76	0.21	0.04984	8	1.919	2
1885	4916.2	4.0	96.97	0.20	0.04992	7	1.921	2
1884	4920.2	4.1	97.17	0.20	0.04999	7	1.923	2
1883	4924.3	4.0	97.37	0.20	0.05006	7	1.925	3
1882	4928.3	4.1	97.57	0.20	0.05013	8	1.928	2
1881	4932.4	4.0	97.77	0.20	0.05021	7	1.930	2
1880	4936.4	4.1	97.97	0.20	0.05028	7	1.932	2
1879	4940.5	4.0	98.17	0.21	0.05035	8	1.934	2
1878	4944.5	4.1	98.38	0.20	0.05043	7	1.936	2
1877	4948.6	4.0	98.58	0.21	0.05050	8	1.938	3
1876	4952.6	4.1	98.79	0.20	0.05058	7	1.941	2
1875	4956.7	4.0	98.99	0.21	0.05065	8	1.943	2
1874	4960.7	4.1	99.20	0.20	0.05073	7	1.945	2
1873	4964.8	4.0	99.40	0.21	0.05080	8	1.947	2
1872	4968.8	4.1	99.61	0.20	0.05088	7	1.949	2
1871	4972.9	4.0	99.81	0.21	0.05095	8	1.951	2
1870	4976.9	4.1	100.02	0.21	0.05103	7	1.953	2
1869	4981.0	4.0	100.23	0.21	0.05110	8	1.955	2
1868	4985.0	4.1	100.44	0 21	0.05118	7	1.957	2
1867	4989.1	4.0	100.65	0.21	0.05125	8	1.959	3
1866	4993.1	4.1	100.86	0.21	0.05133	7	1.962	2
1865	4997.2	4.1	101.07	0.21	0.05140	8	1.964	2
1864	5001.3	4.1	101.28	0.21	0.05148	7	1.966	2
1863	5005.4	4.1	101.49	0.21	0.05155	8	1.968	3
1862	5009.5	4.1	101.70	0.21	0.05163	7	1.671	2
1861	5013.6	4.1	101.91	0.21	0.05170	8	1.973	2
1860	5017.7	4.1	102.12	0.21	0.05178	7	1.975	2
1859	5021.8	4.0	102.33	0.21	0.05185	8	1.977	2
1858	5025.8	4.1	102.54	0.22	0.05193	8	1.979	3
1857	5029.9	4.1	102.76	0.21	0.05201	7	1.982	2
1856	5034.0	4.1	102.97	0.21	0.05208	8	1.984	2
1855	5038.1	4.1	103.18	0.22	0.05216	8	1.986	2
1854	5042.2	4.1	103.40	0.21	0.05224	7	1.988	2
1853	5046.3	4.1	103.61	0.22	0.05231	8	1.990	3
1852	5050.4	4.1	103.83	0.21	0.05239	8	1.993	2
1851	5054.5	4.1	104.04	0.22	0.05247	7	1.995	2

TABLE I—CONTINUED.

V	S_v	*Diff.*	A_v	*Diff.*	I_v	*Diff.*	T_v	*Diff.*
1850	5058.6	4.1	104.26	0.21	0.05254	8	1.997	2
1849	5062.7	4.2	104.47	0.22	0.05262	8	1.999	2
1848	5066.9	4.1	104.69	0.22	0.05270	8	2.001	3
1847	5071.0	4.1	104.91	0.21	0.05278	7	2.004	2
1846	5075.1	4.1	105.12	0.22	0.05285	8	2.006	2
1845	5079.2	4.2	105.34	0.22	0.05293	8	2.008	2
1844	5083.4	4.1	105.56	0.22	0.05301	8	2.010	2
1843	5087.5	4.1	105.78	0.22	0.05309	7	2.012	2
1842	5091.6	4.1	106.00	0.22	0.05316	8	2.014	3
1841	5095.7	4.1	106.22	0.22	0.05324	8	2.017	2
1840	5099.8	4.1	106.44	0.22	0.05332	8	2.019	2
1839	5103.9	4.1	106.66	0.22	0.05340	8	2.021	2
1838	5108.0	4.2	106.88	0.22	0.05348	8	2.023	3
1837	5112.2	4.1	107.10	0.22	0.05356	8	2.026	2
1836	5116.3	4.2	107.32	0.23	0.05364	7	2.028	2
1835	5120.5	4.1	107.55	0.22	0.05371	8	2.030	2
1834	5124.6	4.1	107.77	0.22	0.05379	8	2.032	3
1833	5128.7	4.2	107.99	0.23	0.05387	8	2.035	2
1832	5132.9	4.1	108.22	0.22	0.05395	8	2.037	2
1831	5137.0	4.2	108.44	0.22	0.05403	8	2.039	3
1830	5141.2	4.1	108.66	0.22	0.05411	8	2.042	2
1829	5145.3	4.2	108.88	0.23	0.05419	8	2.044	2
1828	5149.5	4.2	109.11	0.23	0.05427	8	2.046	2
1827	5153.7	4.1	109.34	0.22	0.05435	8	2.048	3
1826	5157.8	4.2	109.56	0.23	0.05443	9	2.051	2
1825	5162.0	4.1	109.79	0.23	0.05452	8	2.053	2
1824	5166.1	4.2	110.02	0.22	0.05460	8	2.055	2
1823	5170.3	4.2	110.24	0.23	0.05468	8	2.057	3
1822	5174.5	4.2	110.47	0.23	0.05476	8	2.060	2
1821	5178.7	4.2	110.70	0.23	0.05484	8	2.062	2
1820	5182.9	4.1	110.93	0.23	0.05492	8	2.064	2
1819	5187.0	4.2	111.16	0.23	0.05500	8	2.066	3
1818	5191.2	4.2	111.39	0.23	0.05508	8	2.069	2
1817	5195.4	4.2	111.62	0.24	0.05516	9	2.071	2
1816	5199.6	4.2	111.86	0.23	0.05525	8	2.073	2
1815	5203.8	4.2	112.09	0.23	0.05533	8	2.075	3
1814	5208.0	4.2	112.32	0.23	0.05541	8	2.078	2
1813	5212.2	4.2	112.55	0.24	0.05549	9	2.080	2
1812	5216.4	4.2	112.79	0.23	0.05558	8	2.082	2
1811	5220.6	4.2	113.02	0.23	0.05566	8	2.084	3

TABLE I—CONTINUED.

V	S_v	*Diff.*	A_v	*Diff.*	I_v	*Diff.*	T_v	*Diff.*
1810	5224.8	4.2	113.25	0.24	0.05574	8	2.087	2
1809	5229.0	4.2	113.49	0.23	0.05582	8	2.089	3
1808	5233.2	4.2	113.72	0.24	0.05590	9	2.092	2
1807	5237.4	4.2	113.96	0.23	0.05599	8	2.094	2
1806	5241.6	4.2	114.19	0.24	0.05607	8	2.096	3
1805	5245.8	4.3	114.43	0.23	0.05615	9	2.099	2
1804	5250.1	4.2	114.66	0.24	0.05624	8	2.101	3
1803	5254.3	4.2	114.90	0.23	0.05632	8	2.104	2
1802	5258.5	4.3	115.13	0.24	0.05640	9	2.106	3
1801	5262.8	4.2	115.37	0.24	0.05649	8	2.109	2
1800	5267.0	4.2	115.61	0.24	0.05657	8	2.111	2
1799	5271.2	4.2	115.85	0.24	0.05665	9	2.113	2
1798	5275.4	4.3	116.09	0.24	0.05674	8	2.115	3
1797	5279.7	4.2	116.33	0.24	0.05682	8	2.118	2
1796	5283.9	4.2	116.57	0.25	0.05690	9	2.120	2
1795	5288.1	4.3	116.82	0.24	0.05699	8	2.122	3
1794	5292.4	4.2	117.06	0.24	0.05707	9	2.125	2
1793	5296.6	4.2	117.30	0.24	0.05716	8	2.127	2
1792	5300.8	4.3	117.54	0.24	0.05724	9	2.129	3
1791	5305.1	4.2	117.78	0.24	0.05733	8	2.132	2
1790	5309.3	4.2	118.02	0.25	0.05741	9	2.134	2
1789	5313.5	4.3	118.27	0.24	0.05750	9	2.136	3
1788	5317.8	4.2	118.51	0.25	0.05759	8	2.139	2
1787	5322.0	4.3	118.76	0.24	0.05767	9	2.141	2
1786	5326.3	4.2	119.00	0.25	0.05776	9	2.143	3
1785	5330.5	4.3	119.25	0.24	0.05785	8	2.146	2
1784	5334.8	4.2	119.49	0.25	0.05793	9	2.148	3
1783	5339.0	4.3	119.74	0.24	0.05802	8	2.151	2
1782	5343.3	4.2	119.98	0.25	0.05810	9	2.153	3
1781	5347.5	4.3	120.23	0.25	0.05819	8	2.156	2
1780	5351.8	4.3	120.48	0.25	0.05827	9	2.158	2
1779	5356.1	4.2	120.73	0.25	0.05836	9	2.160	3
1778	5360.3	4.3	120.98	0.25	0.05845	8	2.163	2
1777	5364.6	4.2	121.23	0.25	0.05853	9	2.165	2
1776	5368.8	4.3	121.48	0.26	0.05862	9	2.167	3
1775	5373.1	4.3	121.74	0.25	0.05871	8	2.170	2
1774	5377.4	4.2	121.99	0.25	0.05879	9	2.172	3
1773	5381.6	4.3	122.24	0.25	0.05888	9	2.175	2
1772	5385.9	4.3	122.49	0.25	0.05897	8	2.177	3
1771	5390.2	4.3	122.74	0.25	0.05905	9	2.180	2

TABLE I—CONTINUED.

V	S_v	*Diff.*	A_v	*Diff.*	I_v	*Diff.*	T_v	*Diff.*
1770	5394.5	4.3	122.99	0.26	0.05914	9	2.182	2
1769	5398.8	4.3	123.25	0.26	0.05923	8	2.184	3
1768	5403.1	4.3	123.51	0.25	0.05931	9	2.187	2
1767	5407.4	4.3	123.76	0.26	0.05940	9	2.189	3
1766	5411.7	4.3	124.02	0.26	0.05949	9	2.192	2
1765	5416.0	4.3	124.28	0.25	0.05958	9	2.194	3
1764	5420.3	4.3	124.53	0.26	0.05967	9	2.197	2
1763	5424.6	4.4	124.79	0.26	0.05976	9	2.199	3
1762	5429.0	4.3	125.05	0.25	0.05985	9	2.202	2
1761	5433.3	4.3	125.30	0.26	0.05994	9	2.204	3
1760	5437.6	4.3	125.56	0.26	0.06003	9	2.207	2
1759	5441.9	4.3	125.82	0.26	0.06012	9	2.209	2
1758	5446.2	4.4	126.08	0.26	0.06021	9	2.211	3
1757	5450.6	4.3	126.34	0.27	0.06030	9	2.214	2
1756	5454.9	4.3	126.61	0.26	0.06039	10	2.216	3
1755	5459.2	4.4	126.87	0.26	0.06049	9	2.219	2
1754	5463.6	4.3	127.13	0.27	0.06058	9	2.221	3
1753	5467.9	4.3	127.40	0.26	0.06067	9	2.224	2
1752	5472.2	4.4	127.66	0.27	0.06076	9	2.226	3
1751	5476.6	4.3	127.93	0.26	0.06085	9	2.229	2
1750	5480.9	4.4	128.19	0.27	0.06094	9	2.231	3
1749	5485.3	4.3	128.46	0.27	0.06103	9	2.234	2
1748	5489.6	4.4	128.73	0.26	0.06112	9	2.236	3
1747	5494.0	4.3	128.99	0.27	0.06121	10	2.239	2
1746	5498.3	4.4	129.26	0.27	0.06131	9	2.241	3
1745	5502.7	4.3	129.53	0.26	0.06140	9	2.244	2
1744	5507.0	4.4	129.79	0.27	0.06149	10	2.246	3
1743	5511.4	4.3	130.06	0.27	0.06159	9	2.249	2
1742	5515.7	4.4	130.33	0.26	0.06168	9	2.251	3
1741	5520.1	4.3	130.59	0.27	0.06177	9	2.254	2
1740	5524.4	4.4	130.86	0.27	0.06186	9	2.256	3
1739	5528.8	4.3	131.13	0.27	0.06195	9	2.259	3
1738	5533.1	4.4	131.40	0.27	0.06204	10	2.262	2
1737	5537.5	4.3	131.67	0.27	0.06214	9	2.264	3
1736	5541.8	4.4	131.94	0.27	0.06223	9	2.267	2
1735	5546.2	4.4	132.21	0.27	0.06232	10	2.269	3
1734	5550.6	4.4	132.48	0.28	0.06242	9	2.272	3
1733	5555.0	4.4	132.76	0.27	0.06251	9	2.275	2
1732	5559.4	4.4	133.03	0.28	0.06260	10	2.277	3
1731	5563.8	4.4	133.31	0.28	0.06270	9	2.280	2

TABLE I—CONTINUED.

V	S_v	*Diff.*	A_v	*Diff.*	I_v	*Diff.*	T_v	*Diff.*
1730	5568.2	4.4	133.59	0.27	0.06279	10	2.282	3
1729	5572.6	4.4	133.86	0.28	0.06289	9	2.285	3
1728	5577.0	4.4	134.14	0.28	0.06297	10	2.288	2
1727	5581.4	4.4	134.42	0.28	0.06307	10	2.290	3
1726	5585.8	4.5	134.70	0.28	0.06317	9	2.293	3
1725	5590.3	4.4	134.98	0.28	0.02326	10	2.296	2
1724	5594.7	4 4	135.26	0.28	0.06336	10	2.298	3
1723	5599.1	4.4	135.54	0.28	0.06346	9	2.301	2
1722	5603.5	4.5	135.82	0.28	0.06355	10	2.303	3
1721	5608.0	4.4	136.10	0.28	0.06365	10	2.306	2
1720	5612.4	4.4	136.38	0.28	0.06375	10	2.308	2
1719	5616.8	4.4	136.66	0.29	0.06385	10	2.310	3
1718	5621.2	4.5	136.95	0.28	0.06395	9	2.313	2
1717	5625.7	4.4	137.23	0.29	0.06404	10	2.315	3
1716	5630.1	4.4	137.52	0.28	0.06414	10	2.318	2
1715	5634.5	4.5	137.80	0.29	0.06424	9	2.320	3
1714	5639.0	4.4	138.09	0.28	0.06433	10	2.323	2
1713	5643.4	4.4	138.37	0.29	0.06443	10	2.325	3
1712	5647.8	4.5	138.66	0.28	0.06453	9	2.328	2
1711	5652.3	4.4	138.94	0.29	0.06462	10	2.330	3
1710	5656.7	4.5	139.23	0.29	0.06472	10	2.333	3
1709	5661.2	4.4	139.52	0.29	0.06482	10	2.336	2
1708	5665.6	4.5	139.81	0.29	0.06492	10	2.338	3
1707	5670.1	4.5	140.10	0.29	0.06502	9	2.341	3
1706	5674.6	4.4	140.39	0.29	0.06511	10	2.344	2
1705	5679.0	4.5	140.68	0.29	0.06521	10	2.346	3
1704	5683.5	4.5	140.97	0.29	0.06531	9	2.349	2
1703	5688.0	4.4	141.26	0.29	0.06540	10	2.351	3
1702	5692.4	4.5	141.55	0.29	0.06550	10	2.354	3
1701	5696.9	4.4	141.84	0.29	0.06560	10	2.357	2
1700	5701.3	4.5	142.13	0.29	0.06570	10	2.359	2
1699	5705.8	4.4	142.42	0.29	0.06580	10	2.361	3
1698	5710.2	4.5	142.71	0.29	0.06590	10	2.364	3
1697	5714.7	4.4	143.00	0.29	0.06600	10	2.367	2
1696	5719.1	4.5	143.29	0.30	0.06610	10	2.369	3
1695	5723.6	4.5	143.59	0.30	0.06620	11	2.372	3
1694	5728.1	4.5	143.89	0.30	0.06631	10	2.375	2
1693	5732.6	4.5	144.19	0.30	0.06641	10	2.377	3
1692	5737.1	4.5	144.49	0.30	0.06651	10	2.380	3
1691	5741.6	4.5	144.79	0.30	0.06661	10	2.383	2

TABLE I—CONTINUED.

V	S_v	*Diff.*	A_v	*Diff.*	I_v	*Diff.*	T_v	*Diff.*
1690	5746.1	4.5	145.09	0.30	0.06671	10	2.385	3
1689	5750.6	4.5	145.39	0.30	0.06681	10	2.388	3
1688	5755.1	4.5	145.69	0.30	0.06691	11	2.391	2
1687	5759.6	4.5	145.99	0.30	0.06702	10	2.393	3
1686	5764.1	4.5	146.29	0.30	0.06712	10	2.396	3
1685	5768.6	4.5	146.59	0.30	0.06722	10	2.399	2
1684	5773.1	4.5	146.89	0.30	0.06732	11	2.401	3
1683	5777.6	4.6	147.19	0.31	0.06743	10	2.404	3
1682	5782.2	4.5	147.50	0.31	0.06753	10	2.407	2
1681	5786.7	4.5	147.81	0.31	0.06763	10	2.409	3
1680	5791.2	4.5	148.12	0.31	0.06773	10	2.412	3
1679	5795.7	4.5	148.43	0.31	0.06783	11	2.415	3
1678	5800.2	4.6	148.74	0.31	0.06794	10	2.418	2
1677	5704.8	4.5	149.05	0.31	0.06804	11	2.420	3
1676	5809.3	4.5	149.36	0.31	0.06815	10	2.423	3
1675	5813.8	4.6	149.67	0.31	0.06825	10	2.426	2
1674	5818.4	4.5	149.98	0.31	0.06835	11	2.428	3
1673	5822.9	4.5	150.29	0.31	0.06846	10	2.431	3
1672	5827.4	4.6	150.60	0.31	0.06856	11	2.434	2
1671	5832.0	4.5	150.91	0.31	0.06867	10	2.436	3
1670	5836.5	4.5	151.22	0.31	0.06877	11	2.439	3
1669	5841.0	4.6	151.53	0.31	0.06888	10	2.442	3
1668	5845.6	4.5	151.84	0.31	0.06898	11	2.445	3
1667	5850.1	4.6	152.15	0.32	0.06909	10	2.448	2
1666	5854.7	4.6	152.47	0.32	0.06919	11	2.450	3
1665	5859.3	4.5	152.79	0.32	0.06930	11	2.453	3
1664	5863.8	4.6	153.11	0.32	0.06941	10	2.456	3
1663	5868.4	4.6	153.43	0.32	0.06951	11	2.459	2
1662	5873.0	4.6	153.75	0.32	0.06962	10	2.461	3
1661	5877.6	4.6	154.07	0.32	0.06972	11	2.464	3
1660	5882.2	4.5	154.39	0.32	0.06983	11	2.467	3
1659	5886.7	4.6	154.71	0.32	0.06994	11	2.470	2
1658	5891.3	4.6	155.03	0.32	0.07005	10	2.472	3
1657	5895.9	4.6	155.35	0.32	0.07015	11	2.475	3
1656	5900.5	4.6	155.67	0.32	0.07026	11	2.478	3
1655	5905.1	4.6	155.99	0.32	0.07037	10	2.481	2
1654	5909.7	4.6	156.31	0.33	0.07047	11	2.483	3
1653	5914.3	4.6	156.64	0.33	0.07058	11	2.486	3
1652	5918.9	4.6	156.97	0.33	0.07069	10	2.489	3
1651	5923.5	4.6	157.30	0.33	0.07079	11	2.492	3

TABLE I—Continued.

V	S_v	Diff.	A_v	Diff.	I_v	Diff.	T_v	Diff.
1650	5928.1	4.6	157.63	0.33	0.07090	11	2.495	3
1649	5932.7	4.6	157.96	0.33	0.07101	11	2.498	3
1648	5937.3	4.6	158.29	0.33	0.07112	11	2.501	2
1647	5941.9	4.6	158.62	0.33	0.07123	11	2.503	3
1646	5946.5	4.6	158.95	0.33	0.07134	11	2.506	3
1645	5951.1	4.6	159.28	0.33	0.07145	11	2.509	3
1644	5955.7	4.7	159.61	0.33	0.07156	11	2.512	3
1643	5960.4	4.6	159.94	0.33	0.07167	11	2.515	2
1642	5965.0	4.6	160.27	0.33	0.07178	11	2.517	3
1641	5969.6	4.7	160.60	0.34	0.07189	11	2.520	3
1640	5974.3	4.6	160.94	0.33	0.07200	11	2.523	3
1639	5978.9	4.7	161.27	0.34	0.07211	11	2.526	3
1638	5983.6	4.6	161.61	0.33	0.07222	11	2.529	3
1637	5988.2	4.6	161.94	0.34	0.07233	12	2.532	2
1636	5992.8	4.7	162.28	0.34	0.07245	11	2.534	3
1635	5997.5	4.6	162.62	0.34	0.07256	11	2.537	3
1634	6002.1	4.6	162.96	0.34	0.07267	11	2.540	3
1633	6006.7	4.7	163.30	0.34	0.07278	12	2.543	3
1632	6011.4	4.6	163.64	0.34	0.07290	11	2.546	3
1631	6016.0	4.7	163.98	0.34	0.07301	11	2.549	3
1630	6020.7	4.6	164.32	0.34	0.07312	11	2.552	3
1629	6025.3	4.7	164.66	0.34	0.07323	11	2.555	3
1628	6030.0	4.6	165.00	0.34	0.07334	12	2.558	2
1627	6034.6	4.7	165.34	0.34	0.07346	11	2.560	3
1626	6039.3	4.7	165.68	0.34	0.07357	12	2.563	3
1625	6044.0	4.7	166.02	0.35	0.07369	11	2.566	3
1624	6048.7	4.7	166.37	0.35	0.07380	12	2.569	3
1623	6053.4	4.7	166.72	0.35	0.07392	11	2.572	2
1622	6058.1	4.7	167.07	0.35	0.07403	11	2.574	3
1621	6062.8	4.7	167.42	0.35	0.07414	12	2.577	3
1620	6067.5	4.7	167.77	0.35	0.07426	12	2.580	3
1619	6072.2	4.7	168.12	0.35	0.07438	11	2.583	3
1618	6076.9	4.7	168.47	0.35	0.07449	12	2.586	2
1617	6081.6	4.7	168.82	0.35	0.07461	12	2.588	3
1616	6086.3	4.7	169.17	0.36	0.07473	11	2.591	3
1615	6091.0	4.7	169.53	0.35	0.07484	12	2.594	3
1614	6095.7	4.7	169.88	0.35	0.07496	11	2.597	3
1613	6100.4	4.7	170.23	0.35	0.07507	12	2.600	3
1612	6105.1	4.8	170.58	0.36	0.07519	11	2.603	3
1611	6109.9	4.7	170.94	0.35	0.07530	12	2.606	3

TABLE I—CONTINUED.

V	S_v	*Diff.*	A_v	*Diff.*	I_v	*Diff.*	T_v	*Diff.*
1610	6114.6	4.7	171.29	0.35	0.07542	12	2.609	3
1609	6119.3	4.7	171.64	0.36	0.07554	11	2.612	2
1608	6124.0	4.7	172.00	0.36	0.07565	12	2.614	3
1607	6128.7	4.8	172.36	0.36	0.07577	12	2.617	3
1606	6133.5	4.7	172.72	0.36	0.07589	12	2.620	3
1605	6138.2	4.7	173.08	0.36	0.07601	12	2.623	3
1604	6142.9	4.8	173.44	0.36	0.07613	11	2.626	3
1603	6147.7	4.7	173.80	0.36	0.07624	12	2.629	3
1602	6152.4	4.7	174.16	0.36	0.07636	12	2.632	3
1601	6157.1	4.7	174.52	0.36	0.07648	12	2.635	3
1600	6161.8	4.7	174.88	0.36	0.07660	12	2.638	3
1599	6166.5	4.8	175.24	0.36	0.07672	12	2.641	3
1598	6171.3	4.7	175.60	0.36	0.07684	12	2.644	3
1597	6176.0	4.8	175.96	0.37	0.07696	12	2.647	3
1596	6180.8	4.8	176.33	0.37	0.07708	12	2.650	3
1595	6185.6	4.7	176.70	0.37	0.07720	12	2.653	3
1594	6190.3	4.8	177.07	0.37	0.07732	13	2.156	3
1593	6195.1	4.8	177.44	0.37	0.07745	12	2.659	3
1592	6199.9	4.8	177.81	0.37	0.07757	12	2.662	3
1591	6004.7	4.8	178.18	0.37	0.07769	12	2.665	3
1590	6209.5	4.7	178.55	0.37	0.07781	12	2.668	3
1589	6214.2	4.8	178.92	0.37	0.07793	12	2.671	3
1588	6219.0	4.8	179.29	0.37	0.07805	13	2.674	3
1587	6223.8	4.8	179.66	0.37	0.07818	12	2.677	3
1586	6228.6	4.8	180.03	0.38	0.07830	12	2.680	4
1585	6233.4	4.8	180.41	0.38	0.07842	12	2.684	3
1584	6238.2	4.8	180.79	0.38	0.07854	13	2.687	3
1583	6243.0	4.8	181.17	0.38	0.07867	12	2.690	3
1582	6247.8	4.9	181.55	0.38	0.07879	12	2.693	3
1581	6252.7	4.8	181.93	0.38	0.07891	12	2.696	3
1580	6257.5	4.8	182.31	0.38	0.07903	12	2.699	3
1579	6262.3	4.8	182.69	0.38	0.07915	13	2.702	3
1578	6267.1	4.9	183.07	0.38	0.07928	12	2.705	3
1577	6272.0	4.8	183.45	0.38	0.07940	13	2.708	3
1576	6276.8	4.8	183.83	0.38	0.07953	12	2.711	3
1575	6281.6	4.8	184.21	0.38	0.07965	13	2.714	3
1574	7286.4	4.9	184.59	0.39	0.07978	12	2.717	3
1573	6291.3	4.8	184.98	0.39	0.07990	13	2.720	3
1572	6296.1	4.9	185.37	0.39	0.08003	12	2.723	3
1571	6301.0	4.8	185.76	0.39	0.08015	13	2.726	3

TABLE I—CONTINUED.

V	S_v	*Diff.*	A_v	*Diff.*	I_v	*Diff.*	T_v	*Diff.*
1570	6305.8	4.8	186.15	0.39	0.08028	13	2.729	3
1569	6310.6	4.9	186.54	0.39	0.08041	12	2.732	3
1568	6315.5	4.8	186.93	0.39	0.08053	13	2.735	3
1567	6320.3	4.8	187.32	0.39	0.08066	13	2.738	3
1566	6325.1	4.9	187.71	0.39	0.08079	13	2.741	4
1565	6330.0	4.8	188.10	0.39	0.08092	12	2.745	3
1564	6334.8	4.9	188.49	0.39	0.08104	13	2.748	3
1563	6339.7	4.8	188.88	0.40	0.08117	13	2.751	3
1562	6344.5	4.9	189.28	0.40	0.08130	13	2.754	3
1561	6349.4	4.8	189.68	0.40	0.08143	13	2.757	3
1560	6354.2	4.9	190.08	0.40	0.08156	13	2.760	3
1559	6359.1	4.8	190.48	0.40	0.08169	13	2.763	3
1558	6363.9	4.9	190.88	0.40	0.08182	13	2.766	3
1557	6368.8	4.9	191.28	0.40	0.08195	13	2.769	4
1556	6373.7	4.9	191.68	0.40	0.08208	13	2.773	3
1555	6378.6	4.9	192.08	0.40	0.08221	13	2.776	3
1554	6383.5	4.9	192.48	0.40	0.08234	13	2.779	3
1553	6388.4	4.9	192.88	0.40	0.08247	13	2.782	3
1552	6393.3	4.9	193.28	0.41	0.08260	13	2.785	4
1551	6398.2	4.9	193.69	0.41	0.08273	13	2.789	3
1550	6403.1	4.9	194.10	0.41	0.08286	13	2.792	3
1549	6408.0	4.9	194.51	0.41	0.08299	13	2.795	3
1548	6412.9	4.9	194.92	0.41	0.08312	14	2.798	3
1547	6417.8	4.9	195.33	0.41	0.08326	13	2.801	4
1546	6422.7	5.0	195.74	0.41	0.08339	13	2.805	3
1545	6427.7	4.9	196.15	0.41	0.08352	14	2.808	3
1544	6432.6	5.0	196.56	0.41	0.08366	13	2.811	3
1543	6437.6	4.9	196.97	0.41	0.08379	13	2.814	4
1542	6442.5	5.0	197.38	0.41	0.08392	14	2.818	3
1541	6447.5	4.9	197.79	0.42	0.08406	13	2.821	3
1540	6452.4	5.0	198.21	0.42	0.08419	13	2.824	3
1539	6457.4	4.9	198.63	0.42	0.08432	14	2.827	3
1538	6462.3	4.9	199.05	0.42	0.08446	13	2.830	4
1537	6467.2	4.9	199.47	0.42	0.08459	14	2.834	3
1536	6472.1	5.0	199.89	0.42	0.08473	13	2.837	3
1535	6477.1	4.9	200.31	0.42	0.08486	14	2.840	3
1534	6482.0	5.0	200.73	0.42	0.08500	13	2.843	3
1533	6487.0	4.9	201.15	0.42	0.08513	14	2.846	4
1532	6491.9	5.0	201.57	0.42	0.08527	13	2.850	3
1531	6496.9	4.9	201.99	0.42	0.08540	14	2.853	3

TABLE I—Continued.

V	S_v	Diff.	A_v	Diff.	I_v	Diff.	T_v	Diff.
1530	6501.8	5.0	202.41	0.43	0.08554	14	2.856	3
1529	6506.8	4.9	202.84	0.43	0.08568	14	2.859	3
1528	6511.7	5.0	203.27	0.43	0.08582	13	2.862	4
1527	6516.7	5.0	203.70	0.43	0.08595	14	2.866	3
1526	6521.7	5.0	204.13	0.43	0.08609	14	2.869	3
1525	6526.7	5.0	204.56	0.43	0.08623	13	2.872	4
1524	6531.7	5.0	204.99	0.43	0.08636	14	2.876	3
1523	6536.7	5.0	205.42	0.43	0.08650	14	2.879	3
1522	6541.7	5.0	205.85	0.43	0.08664	13	2.882	4
1521	6546.7	5.0	206.28	0.43	0.18677	14	2.886	3
1520	6551.7	5.0	206.71	0.43	0 08691	14	2.889	3
1519	6556.7	5.0	207.14	0.44	0.08705	14	2.892	3
1518	6561.7	5.0	207.58	0.44	0.08719	14	2.895	4
1517	6566.7	5.0	208.02	0.44	0.08733	14	2.899	3
1516	6571.7	5.0	208.46	0.44	0.08747	15	2.902	3
1515	6576.7	5.0	208.90	0.44	0.08762	14	2.905	4
1514	6581.7	5.0	209.34	0.44	0.08776	14	2.909	3
1513	6586.7	5.1	209.78	0.44	0.08790	14	2.912	3
1512	6591.8	5.0	210.22	0.44	0.08804	14	2.915	4
1511	6596.8	5.0	210.66	0.44	0.08818	14	2.919	3
1510	6601.8	5.0	211.10	0.44	0.08832	14	2.922	3
1509	6606.8	5.1	211.54	0.45	0.08846	14	2.925	3
1508	6611.9	5.0	211.99	0.45	0.08860	15	2.928	4
1507	6616.9	5.1	212.44	0.45	0.08875	14	2.932	3
1506	6622.0	5.0	212.89	0.45	0.08889	14	2.935	3
1505	6627.0	5.1	213.34	0.45	0.08903	15	2.938	4
1504	6632.1	5.0	213.79	0.45	0.08918	14	2.942	3
1503	6637.1	5.1	214.24	0.45	0.08932	14	2.945	3
1502	6642.2	5.0	214.69	0.45	0.08946	15	2.948	4
1501	6647.2	5.1	215.14	0.45	0.08961	14	2.952	3
1500	6652.3	5.0	215.59	0.46	0.08975	14	2.955	3
1499	6657.3	5.1	216.05	0.46	0.08989	15	2.958	4
1498	6662.4	5.0	216.51	0.46	0.09004	15	2.962	3
1497	6667.4	5.1	216.97	0.46	0.09019	14	2.965	3
1496	6672.5	5.1	217.43	0.46	0.09033	15	2.968	4
1495	6677.6	5.1	217.89	0.46	0.09048	15	2.972	3
1494	6682.7	5.1	218.35	0.46	0.09063	14	2.975	4
1493	6687.8	5.1	218.81	0.46	0.09077	15	2.979	3
1492	6692.9	5.1	219.27	0.46	0.09092	15	2.982	3
1491	6698.0	5.1	219.73	0.46	0.09107	15	2.985	4

TABLE I—Continued.

V	S_v	Diff.	A_v	Diff.	I_v	Diff.	T_v	Diff.
1490	6703.1	5.1	220.19	0.47	0.09122	15	2.989	3
1489	6708.2	5.1	220.66	0.47	0.09137	14	2.992	4
1488	6713.3	5.1	221.13	0.47	0.09151	15	2.996	3
1487	6718.4	5.1	221.60	0.47	0.09166	15	2.999	4
1486	6723.5	5.1	222.07	0.47	0.09181	15	3.003	3
1485	6728.6	5.1	222.54	0.47	0.09196	15	3.006	4
1484	6733.7	5.1	223.01	0.47	0.09211	15	3.010	3
1483	6738.8	5.2	223.48	0.47	0.09226	15	3.013	4
1482	6744.0	5.1	223.95	0.47	0.09241	15	3.017	3
1481	6849.1	5.2	224.42	0.48	0.09256	15	3.020	4
1480	6754.3	5.1	224.90	0.48	0.09271	15	3.024	3
1479	6759.4	5.2	225.38	0.48	0.09286	15	3.027	4
1478	6764.6	5.1	225.86	0.48	0.09301	16	3.031	3
1477	6769.7	5.2	226.34	0.48	0.09317	15	3.034	4
1476	6774.9	5.1	226.82	0.48	0.09332	15	3.038	3
1475	6780.0	5.2	227.30	0.48	0.09347	15	3.041	4
1474	6785.2	5.1	227.78	0.48	0.09362	16	3.045	3
1473	6790.3	5.2	228.26	0.48	0.09378	15	3.048	4
1472	6795.5	5.1	228.74	0.49	0.09393	15	3.052	3
1471	6800.6	5.2	229.23	0.49	0.09408	15	3.055	4
1470	6805.8	5.1	229.72	0.49	0.09423	15	3.059	3
1469	6810.9	5.2	230.21	0.49	0.09438	16	3.062	4
1468	6816.1	5.2	230.70	0.49	0.06454	16	3.066	3
1467	6821.3	5.2	231.19	0.49	0.09470	15	3.069	4
1466	6826.5	5.2	231.68	0.49	0.09485	16	3.073	3
1465	6831.7	5.2	232.17	0.49	0.09501	16	3.076	4
1464	6836.9	5.2	232.66	0.49	0.09517	15	3.080	3
1463	6842.1	5.2	233.15	0.50	0.09532	16	3.083	4
1462	6847.3	5.2	233.65	0.50	0.09548	16	3.087	3
1461	6852.5	5.2	234.15	0.50	0.09564	15	3.090	4
1460	6857.7	5.2	234.65	0.50	0.09579	16	3.094	3
1459	6862.9	5.2	235.15	0.50	0.09595	16	3.097	4
1458	6868.1	5.3	235.65	0.50	0.09611	16	3.101	4
1457	6873.4	5.2	236.15	0.50	0.09627	15	3.105	3
1456	6878.6	5.2	236.65	0.50	0.09642	16	3.108	4
1455	6883.8	5.3	237.15	0.51	0.09658	16	3.112	4
1454	6889.1	5.2	237.66	0.51	0.09674	16	3.116	3
1453	6894.3	5.2	238.17	0.51	0.09690	15	3.119	4
1452	6899.5	5.3	238.68	0.51	0.09705	16	3.123	4
1451	6904.8	5.2	239.19	0.51	0.09721	16	3.127	3

TABLE I—CONTINUED.

V	S_v	Diff.	A_v	Diff.	I_v	Diff.	T_v	Diff.
1450	6910.0	5.3	239.70	0.51	0.09737	16	3.130	4
1449	6915.3	5.2	240.21	0.51	0.09753	16	3.134	4
1448	6920.5	5.3	240.72	0.51	0.09769	16	3.138	3
1447	6925.8	5.2	241.23	0.52	0.09785	17	3.141	4
1446	6931.0	5.3	241.75	0.52	0.09802	16	3.145	3
1445	6936.3	5.2	242.27	0.52	0.09818	16	3.148	4
1444	6941.5	5.3	242.79	0.52	0.09834	16	3.152	4
1443	6946.8	5.2	243.31	0.52	0.09850	16	3.156	3
1442	6952.0	5.3	243.83	0.52	0.09866	17	3.159	4
1441	6957.3	5.2	244.35	0.52	0.09883	16	3.163	3
1440	6962.5	5.3	244.87	0.52	0.09899	17	3.166	4
1439	6967.8	5.3	245.39	0.53	0.09916	17	3.170	4
1438	6973.1	5.3	245.92	0.53	0.09933	16	3.174	3
1437	6978.4	5.3	246.45	0.53	0.09949	17	3.177	4
1436	6983.7	5.3	246.98	0.53	0.09966	16	3.181	4
1435	6989.0	5.3	247.51	0.53	0.09982	17	3.185	3
1434	6994.3	5.3	248.04	0.53	0.09999	16	3.188	4
1433	6999.6	5.3	248.57	0.53	0.10015	17	3.192	4
1432	7004.9	5.3	249.10	0.53	0.10032	16	3.196	3
1431	7010.2	5.3	249.63	0.53	0.10048	17	3.199	4
1430	7015.5	5.3	250.16	0.54	0.10065	17	3.203	4
1429	7020.8	5.4	250.70	0.54	0.10082	16	3.207	4
1428	7026.2	5.3	261.24	0.54	0.10098	17	3.211	3
1427	7031.5	5.3	251.78	0.54	0.10115	17	3.214	4
1426	7036.8	5.4	252.32	0.54	0.10132	17	3.218	4
1425	7042.2	5.3	252.86	0.54	0.10149	17	3.222	3
1424	7047.5	5.3	253.40	0.54	0.10166	17	3.225	4
1423	7052.8	5.4	253.94	0.54	0.10183	17	3.229	4
1422	7058.2	5.3	254.48	0.54	0.10200	17	3.233	3
1421	7063.5	5.4	255.02	0.55	0.10217	17	3.236	4
1420	7068.9	5.3	255.57	0.55	0.10234	17	3.240	4
1419	7074.2	5.4	256.12	0.55	0.10251	17	3.244	4
1418	7079.6	5.4	256.67	0.55	0.10268	17	3.248	4
1417	7085.0	5.3	257.22	0.55	0.10285	18	3.252	3
1416	7090.3	5.4	257.77	0.55	0.10303	17	3.255	4
1415	7095.7	5.3	258.32	0.55	0.10320	17	3.259	4
1414	7101.0	5.4	258.87	0.55	0.10337	17	3.263	4
1413	7106.4	5.3	259.42	0.56	0.10354	18	3.267	3
1412	7111.7	5.4	259.98	0.56	0.10372	17	3.270	4
1411	7117.1	5.4	260.54	0.56	0.10389	17	3.274	4

TABLE I—CONTINUED.

V	S_v	*Diff.*	A_v	*Diff.*	I_v	*Diff.*	T_v	*Diff.*
1410	7122.5	5.4	261.10	0.56	0.10406	18	3.278	4
1409	7127.9	5.4	261.66	0.56	0.10424	18	3.282	4
1408	7133.3	5.4	262.22	0.56	0.10442	17	3.286	3
1407	7138.7	5.4	262.78	0.56	0.10459	18	3.289	4
1406	7144.1	5.5	263.34	0.57	0.10477	18	3.293	4
1405	7149.6	5.4	263.91	0.57	0.10495	17	3.297	4
1404	7155.0	5.4	264.48	0.57	0.10512	18	3.301	4
1403	7160.4	5.4	265.05	0.57	0.10530	18	3.305	4
1402	7165.8	5.4	265.62	0.57	0.10548	17	3.309	4
1401	7171.2	5.4	266.19	0.57	0.10565	18	3.313	4
1400	7176.6	5.4	266.76	0.58	0.10583	18	3.317	4
1399	7182.0	5.5	267.34	0.58	0.10601	18	3.321	3
1398	9187.5	5.4	267.92	0.58	0.10619	18	3.324	4
1397	7192.9	5.5	268.50	0.58	0.10637	18	3.328	4
1396	7198.4	5.4	269.08	0.58	0.10655	18	3.332	4
1395	7203.8	5.5	269.66	0.58	0.10673	18	3.336	4
1394	7209.3	5.4	270.24	0.58	0.10691	18	3.340	4
1393	7214.7	5.5	270.82	0.58	0.10709	18	3.344	4
1392	7220.2	5.4	271.40	0.58	0.10727	18	3.348	4
1391	7225.6	5.5	271.98	0.58	0.10745	18	3.352	4
1390	7231.1	5.4	272.56	0.59	0.10763	18	3.356	4
1389	7236.5	5.5	273.15	0.59	0.10781	18	3.360	4
1388	7242.0	5.5	273.74	0.59	0.10799	19	2.364	4
1387	7247.5	5.5	274.33	0.59	0.10818	18	3.368	4
1386	7253.0	5.5	274.92	0.59	0.10836	19	3.372	4
1385	7258.5	5.5	275.51	0.60	0.10855	18	3.376	4
1384	7264.0	5.5	276.11	0.60	0.10873	18	3.380	4
1383	7269.5	5.5	276.71	0.60	0.10891	19	3.384	4
1382	7275.0	5.5	277.31	0.60	0.10910	18	3.388	4
1381	7280.5	5.5	277.91	0.60	0.10928	19	3.392	4
1380	7286.0	5.5	278.51	0.61	0.10947	19	3.396	4
1379	7291.5	5.5	279.12	0.61	0.10966	18	3.400	4
1378	7297.0	5.6	279.73	0.61	0.10984	19	3.404	4
1377	7302.6	5.5	280.34	0.61	0.11003	19	3.408	4
1376	7308.1	5.5	280.95	0.62	0.11022	18	3.412	4
1375	7313.6	5.6	281.57	0.61	0.11040	19	3.416	4
1374	7319.2	5.5	282.18	0.61	0.11059	19	3.420	4
1373	7324.7	5.5	282.79	0.61	0.11078	19	3.424	4
1372	7330.2	5.6	283.40	0.62	0.11097	19	3.428	4
1371	7335.8	5.5	284.02	0.62	0.11116	19	3.432	4

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TABLE I—CONTINUED.

V	S_v	*Diff.*	A_v	*Diff.*	I_v	*Diff.*	T_v	*Diff.*
1370	7341.3	5.6	284.64	0.62	0.11135	19	3.436	4
1369	7346.9	5.6	285.26	0.62	0.11154	19	3.440	4
1368	7352.5	5.6	285.88	0.62	0.11173	19	3.444	4
1367	7358.1	5.6	286.50	0.63	0.11192	19	3.448	5
1366	7363.7	5.6	287.13	0.63	0.11211	19	3.453	4
1365	7369.3	5.6	287.76	0.63	0.11230	19	3.457	4
1364	7374.9	5.6	288.39	0.63	0.11249	19	3.461	4
1363	7380.5	5.7	289.02	0.64	0.11268	20	3.465	4
1362	7386.2	5.6	289.66	0.64	0.11288	20	3.469	4
1361	7391.8	5.6	290.30	0.64	0.11308	20	3.473	4
1360	7397.4	5.6	290.94	0.64	0.11328	20	3.477	4
1359	7403.0	5.7	291.58	0.64	0.11348	20	3.481	4
1358	7408.7	5.7	292.22	0.65	0.11368	20	3.485	4
1357	7414.4	5.7	292.87	0.65	0.11388	20	3.489	5
1356	7420.1	5.7	293.52	0.65	0.11408	20	3.494	4
1355	7425.8	5.7	294.17	0.65	0.11428	20	3.498	4
1354	7431.5	5.7	294.82	0.65	0.11448	20	3.502	4
1353	7437.2	5.7	295.47	0.65	0.11468	20	3.506	5
1352	7442.9	5.7	296.12	0.66	0.11488	20	3.511	4
1351	7448.6	5.7	296.78	0.66	0.11508	20	3.515	4
1350	7454.3	5.7	297.44	0.66	0.11528	20	3.519	4
1349	7460.0	5.8	298.10	0.66	0.11548	20	3.523	4
1348	7465.8	5.7	298.76	0.67	0.11568	20	3.527	5
1347	7471.5	5.8	299.43	0.67	0.11588	20	3.532	4
1346	7477.3	5.8	300.10	0.67	0.11608	20	3.536	4
1345	7483.1	5.8	300.77	0.67	0.11628	21	3.540	5
1344	7488.9	5.8	301.44	0.68	0.11649	21	3.545	4
1343	7494.7	5.8	302.12	0.68	0.11670	21	3.549	4
1342	7500.5	5.8	302.80	0.68	0.11691	21	3.553	5
1341	7506.3	5.8	303.48	0.68	0.11712	21	3.558	4
1340	7512.1	5.8	304.16	0.68	0.11733	21	3.562	4
1339	7517.9	5.8	304.84	0.69	0.11754	21	3.566	5
1338	7523.7	5.9	305.53	0.69	0.11775	21	3.571	4
1337	7529.6	5.8	306.22	0.69	0.11796	21	3.575	4
1336	7535.4	5.9	306.91	0.69	0.11817	21	3.579	5
1335	7541.3	5.9	307.60	0.70	0.11838	21	3.584	4
1334	7547.2	5.8	308.30	0.70	0.11859	21	3.588	5
1333	7553.0	5.9	309.00	0.70	0.11880	21	3.593	4
1332	7558.9	5.9	309.70	0.70	0.11901	22	3.597	5
1331	7564.8	5.9	310.40	0.70	0.11923	22	3.602	4

TABLE I—CONTINUED.

V	*S*.	*Diff.*	*A*.	*Diff.*	*I*.	*Diff.*	*T*.	*Diff.*
1330	7570.7	5.9	311.10	0.71	0.11945	22	3.606	4
1329	7576.6	6.0	311.81	0.71	0.11967	22	3.610	5
1328	7582.6	5.9	312.52	0.71	0.11989	22	3.615	4
1327	7588.5	6.0	313.23	0.72	0.12011	21	3.619	5
1326	7594.5	5.9	313.95	0.72	0.12032	22	3.624	4
1325	7600.4	6.0	314.67	0.72	0.12054	22	3.628	5
1324	7606.4	5.9	315.39	0.72	0.12076	22	3.633	4
1323	7612.3	6.0	316.11	0.72	0.12098	21	3.637	5
1322	7618.3	5.9	316.83	0.72	0.12119	22	3.642	4
1321	7624.2	6.0	317.55	0.73	0.12141	22	3.646	5
1320	7630.2	6.0	318.28	0.73	0.12163	22	3.651	4
1319	7636.2	6.0	319.01	0.73	0.12185	22	3.655	5
1318	7642.2	6.0	319.74	0.73	0.12207	22	3.660	5
1317	7648.2	6.0	320.47	0.74	0.12229	22	3.665	4
1316	7654.2	6.0	321.21	0.74	0.12251	22	3.669	5
1315	7660.2	6.1	321.95	0.74	0.12273	22	3.674	4
1314	7666.3	6.1	322.69	0.75	0.12295	23	3.678	5
1313	7672.4	6.1	323.44	0.75	0.12318	23	3.683	4
1312	7678.5	6.0	324.19	0.75	0.12341	23	3.687	5
1311	7684.5	6.1	324.94	0.76	0.12364	23	3.692	5
1310	7690.6	6.1	325.70	0.76	0.12387	23	3.697	5
1309	7696.7	6.1	326.46	0.76	0.12410	23	3.702	5
1308	7702.8	6.1	327.22	0.76	0.12433	23	3.707	4
1307	7708.9	6.2	327.98	0.76	0.12456	23	3.711	5
1306	7715.1	6.1	328.74	0.77	0.12479	23	3.716	5
1305	7721.2	6.2	329.51	0.77	0.12502	23	3.721	4
1304	7727.4	6.1	330.28	0.77	0.12525	23	3.725	5
1303	7733.5	6.1	331.05	0.77	0.12548	23	3.730	5
1302	7739.6	6.2	331.82	0.78	0.12571	24	3.735	4
1301	7745.8	6.2	332.60	0.78	0.12595	24	3.739	5
1300	7752.0	6.2	333.38	0.78	0.12619	23	3.744	5
1299	7758.2	6.2	334.16	0.78	0.12642	24	3.749	5
1298	7764.4	6.2	334.94	0.79	0.12666	24	3.754	5
1297	7770.6	6.3	335.73	0.79	0.12690	24	3.759	4
1296	7776.9	6.2	336.52	0.79	0.12714	24	3.763	5
1295	7783.1	6.2	337.31	0.79	0.12738	24	3.768	5
1294	7789.3	6.2	338.10	0.80	0.12762	24	3.773	5
1293	7795.5	6.3	338.90	0.80	0.12786	24	3.778	4
1292	7801.8	6.2	339.70	0.80	0.12810	24	3.782	5
1291	7808.0	6.3	340.50	0.80	0.12834	24	3.787	5

TABLE I—CONTINUED.

V	S_v	*Diff.*	A_v	*Diff.*	I_v	*Diff.*	T_v	*Diff.*
1290	7814.3	6.3	341.30	0.81	0.12858	24	3.792	5
1289	7820.6	6.3	342.11	0.81	0.12882	24	3.797	5
1288	7826.9	6.3	342.92	0.81	0.12906	24	3.802	5
1287	7833.2	6.3	343.73	0.82	0.12930	25	3.807	5
1286	7839.5	6.3	344.55	0.82	0.12955	25	3.812	5
1285	7845.8	6.4	345.37	0.82	0.12980	25	3.817	4
1284	7852.2	6.3	346.19	0.83	0.13005	25	3.821	5
1283	7858.5	6.4	347.02	0.83	0.13030	25	3.826	5
1282	7864.9	6.3	347.85	0.83	0.13055	25	3.831	5
1281	7871.2	6.4	348.68	0.84	0.13080	25	3.836	5
1280	7877.6	6.4	349.52	0.84	0.13105	25	3.841	5
1279	7884.0	6.4	350.36	0.84	0.13130	25	3.846	5
1278	7890.4	6.4	351.20	0.84	0.13155	25	3.851	5
1277	7896.8	6.4	352.04	0.85	0.13180	25	3.856	5
1276	7903.2	6.4	352.89	0.85	0.13205	25	3.861	5
1275	7909.6	6.5	353.74	0.85	0.13230	25	3.866	5
1274	7916.1	6.4	354.59	0.86	0.13255	26	3.871	5
1273	7922.5	6.5	355.45	0.86	0.13281	26	3.876	5
1272	7929.0	6.4	356.31	0.86	0.13307	26	3.881	5
1271	7935.4	6.5	357.17	0.86	0.13333	26	3.886	5
1270	7941.9	6.5	358.03	0.87	0.13359	26	3.891	5
1269	7948.4	6.5	358.90	0.87	0.13385	26	3.896	5
1268	7954.9	6.5	359.77	0.87	0.13411	26	3.901	5
1267	7961.4	6.5	360.64	0.87	0.13437	26	3.906	6
1266	7967.9	6.5	361.51	0.88	0.13463	26	3.912	5
1265	7974.4	6.6	362.39	0.88	0.13489	26	3.917	5
1264	7981.0	6.5	363.27	0.89	0.13515	26	3.922	5
1263	7987.5	6.6	364.16	0.89	0.13541	27	3.927	6
1262	7994.1	6.5	365.05	0.89	0.13568	27	3.933	5
1261	8000.6	6.6	365.94	0.90	0.13595	27	3.938	5
1260	8007.2	6.6	366.84	0.90	0.13622	27	3.943	5
1259	8013.8	6.6	367.74	0.90	0.13649	27	3.948	6
1258	8020.4	6.6	368.64	0.90	0.13676	27	3.954	5
1257	8027.0	6.7	369.54	0.91	0.13703	27	3.959	5
1256	8033.7	6.6	370.45	0.91	0.13730	27	3.964	6
1255	8040.3	6.6	371.36	0.91	0.13757	27	3.970	5
1254	8046.9	6.7	372.27	0.92	0.13784	27	3.975	5
1253	8053.6	6.6	373.19	0.92	0.13811	27	3.980	5
1252	8060.2	6.7	374.11	0.92	0.13838	28	3.985	6
1251	8066.9	6.7	375.03	0.93	0.13866	28	3.991	5

TABLE I—CONTINUED.

V	S_v	*Diff.*	A_v	*Diff.*	I_v	*Diff.*	T_v	*Diff.*
1250	8073.6	6.7	375.96	0.93	0.13894	27	3.996	5
1249	8080.3	6.7	376.89	0.93	0.13921	28	4.001	6
1248	8087.0	6.7	377.82	0.94	0.13949	27	4.007	5
1247	8093.7	6.8	378.76	0.94	0.13976	28	4.012	6
1246	8100.5	6.7	379.70	0.95	0.14004	28	4.018	5
1245	8107.2	6.8	380.65	0.95	0.14032	28	4.023	6
1244	8114.0	6.7	381.60	0.95	0.14060	28	4.029	5
1243	8120.7	6.8	382.55	0.96	0.14088	28	4.034	5
1242	8127.5	6.7	383.51	0.96	0.14116	28	4.039	6
1241	8134.2	6.8	384.47	0.96	0.14144	28	4.045	5
1240	8141.0	6.8	385.43	0.97	0.14172	28	4.050	6
1239	8147.8	6.8	386.40	0.97	0.14200	29	4.056	5
1238	8154.6	6.8	387.37	0.97	0.14229	29	4.061	6
1237	8161.4	6.9	388.34	0.98	0.14258	29	4.067	5
1236	8168.3	6.8	389.32	0.98	0.14287	29	4.072	6
1235	8175.1	6.9	390.30	0.98	0.14316	29	4.078	5
1234	8182.0	6.8	391.28	0.98	0.14345	29	4.083	6
1233	8188.8	6.9	392.26	0.99	0.14374	29	4.089	6
1232	8195.7	6.9	393.25	0.99	0.14403	29	4.095	5
1231	8202.6	6.9	394.24	0.99	0.14432	29	4.100	6
1230	3209.5	6.9	395.23	1.00	0.14461	30	4.106	6
1229	8216.4	7.0	396.23	1.00	0.14491	30	4.112	6
1228	8223.4	7.0	397.23	1.01	0.14521	30	4.118	5
1227	8230.4	7.0	398.24	1.01	0.14551	30	4.123	6
1226	8237.4	7.0	399.25	1.02	0.14581	30	4.129	6
1225	8244.4	7.0	400.27	1.03	0.14611	30	4.135	5
1224	8251.4	7.0	401.30	1.04	0.14641	30	4.140	6
1223	8258.4	7.1	402.34	1.04	0.14671	30	4.146	6
1222	8265.5	7.1	403.38	1.05	0.14701	30	4.152	5
1221	8272.6	7.1	404.43	1.05	0.14731	31	4.157	6
1220	8279.7	7.1	405.48	1.06	0.14762	31	4.163	6
1219	8286.8	7.2	406.54	1.06	0.14793	31	4.169	6
1218	8294.0	7.2	407.60	1.07	0.14824	31	4.175	6
1217	8301.2	7.2	408.67	1.07	0.14855	31	4.181	6
1216	8308.4	7.2	409.74	1.08	0.14886	31	4.187	6
1215	8315.6	7.3	410.82	1.08	0.14917	32	4.193	6
1214	8322.9	7.3	411.90	1.09	0.14949	32	4.199	6
1213	8330.2	7.3	412.99	1.09	0.14981	32	4.205	6
1212	8337.5	7.2	414.08	1.10	0.15013	32	4.211	6
1211	8344.7	7.3	415.18	1.10	0.15045	32	4.217	6

TABLE I—CONTINUED.

V	S_v	*Diff.*	A_v	*Diff.*	I_v	*Diff.*	T_v	*Diff.*
1210	8352.0	7.4	416.28	1.11	0.15077	33	4.223	6
1209	8359.4	7.4	417.39	1.12	0.15110	33	4.229	6
1208	8366.8	7.4	418.51	1.13	0.15143	33	4.235	6
1207	8374.2	7.5	419.64	1.13	0.15176	33	4.241	7
1206	8381.7	7.5	420.77	1.14	0.15209	33	4.248	6
1205	8389.2	7.5	421.91	1.14	0.15242	33	4.254	6
1204	8396.7	7.5	423.05	1.15	0.15275	33	4.260	6
1203	8404.2	7.6	424.20	1.16	0.15308	33	4.266	7
1202	8411.8	7.5	425.36	1.16	0.15341	34	4.273	6
1201	8419.3	7.6	426.52	1.17	0.15375	34	4.279	6
1200	8426.9	7.6	427.69	1.18	0.15409	34	4.285	6
1199	8434.5	7.6	428.87	1.18	0.15443	34	4.291	7
1198	8442.1	7.6	430.05	1.19	0.15477	35	4.298	6
1197	8449.7	7.7	431.24	1.20	0.15512	35	4.304	7
1196	8457.4	7.8	432.44	1.20	0.15547	35	4.311	6
1195	8465.2	7.8	433.64	1.21	0.15582	35	4.317	6
1194	8473.0	7.8	434.85	1.22	0.15617	35	4.323	7
1193	8480.8	7.8	436.07	1.22	0.15652	35	4.330	6
1192	8488.6	7.8	437.29	1.23	0.15687	35	4.336	6
1191	8496.4	7.9	438.52	1.23	0.15722	36	4.342	7
1190	8504.3	7.9	439.75	1.24	0.15758	36	4.349	7
1189	8512.2	7.9	440.99	1.25	0.15794	36	4.356	6
1188	8520.1	7.9	442.24	1.26	0.15830	36	4.362	6
1187	8528.0	8.0	443.50	1.27	0.15866	36	4.368	7
1186	8536.0	8.0	444.77	1.28	0.15902	37	4.375	7
1185	8544.0	8.0	446.05	1.28	0.15939	37	4.382	7
1184	8552.0	8.0	447.33	1.29	0.15976	37	4.389	7
1183	8560.0	8.1	448.62	1.29	0.16013	37	4.396	7
1182	8568.1	8.1	449.91	1.30	0.16050	37	4.403	7
1181	8576.2	8.1	451.21	1.30	0.16087	37	4.410	7
1180	8584.3	8.1	452.51	1.31	0.16124	38	4.417	7
1179	8592.4	8.2	453.82	1.32	0.16162	38	4.424	7
1178	8600.6	8.2	455.14	1.33	0.16200	38	4.431	7
1177	8608.8	8.3	456.47	1.34	0.16238	38	4.438	7
1176	8617.1	8.3	457.81	1.34	0.16276	38	4.445	7
1175	8625.4	8.3	459.15	1.35	0.16314	38	4.452	7
1174	8633.7	8.3	460.50	1.36	0.16352	39	4.459	7
1173	8642.0	8.3	461.86	1.36	0.16391	39	4.466	7
1172	8650.3	8.4	463.22	1.37	0.16430	40	4.473	7
1171	8658.7	8.4	464.59	1.38	0.16470	40	4.480	7

TABLE I—CONTINUED.

V	S_v	*Diff.*	A_v	*Diff.*	I_v	*Diff.*	T_v	*Diff.*
1170	8667.1	8.4	465.97	1.39	0.16510	40	4.487	7
1169	8675.5	8.5	467.36	1.41	0.16550	40	4.494	8
1168	8684.0	8.5	468.77	1.41	0.16590	40	4.502	7
1167	8692.5	8.5	470.18	1.42	0.16630	40	4.509	7
1166	8701.0	8.6	471.60	1.43	0.16670	41	4.516	8
1165	8709.6	8.6	473.03	1.44	0.16711	41	4.524	7
1164	8718.2	8.6	474.47	1.45	0.16752	41	4.531	8
1163	8726.8	8.6	475.92	1.46	0.16793	41	4.539	7
1162	8735.4	8.6	977.38	1.47	0.16834	41	4.546	8
1161	8744.0	8.7	478.85	1.49	0.16875	41	4.554	7
1160	8752.7	8.8	480.34	1.50	0.16916	42	4.561	7
1159	8761.5	8.8	481.84	1.50	0.16958	42	4.568	8
1158	8770.3	8.8	483.34	1.51	0.17000	42	4.576	8
1157	8779.1	8.8	484.85	1.51	0.17042	43	4.584	7
1156	8787.9	8.8	486.36	1.52	0.17085	43	4.591	8
1155	8796.7	8.9	487.88	1.52	0.17128	43	4.599	8
1154	8805.6	8.9	489.40	1.53	0.17171	43	4.607	7
1153	8814.5	9.0	490.93	1.53	0.17214	43	4.614	8
1152	8823.5	9.0	492.46	1.53	0.17257	43	4.622	8
1151	8832.5	9.0	493.99	1.54	0.17300	44	4.630	8
1150	8841.5	9.0	495.53	1.56	0.17344	44	4.638	8
1149	8850.5	9.1	497.09	1.57	0.17388	44	4.646	8
1148	8859.6	9.1	498.66	1.58	0.17432	44	4.654	8
1147	8868.7	9.1	500.24	1.59	0.17476	45	4.662	8
1146	8877.8	9.2	501.83	1.60	0.17521	45	4.670	8
1145	8887.0	9.2	503.43	1.62	0.17506	45	4.678	8
1144	8896.2	9.2	505.05	1.64	0.17611	45	4.686	8
1143	8905.4	9.3	506.69	1.65	0.17656	46	4.694	8
1142	8914.7	9.3	508.34	1.66	0.17702	46	4.702	8
1141	8924.0	9.3	510.00	1.67	0.17748	46	4.710	8
1140	8933.3	9.4	511.67	1.67	0.17794	47	4.718	8
1139	8942.7	9.4	513.34	1.68	0.17841	47	4.726	8
1138	8952.1	9.4	515.02	1.69	0.17888	47	4.734	9
1137	8961.5	9.4	516.71	1.70	0.17935	47	4.743	8
1136	8970.9	9.5	518.41	1.71	0.17982	47	4.751	9
1135	8980.4	9.5	520.12	1.71	0.18029	48	4.760	8
1134	8989.9	9.6	521.83	1.73	0.18077	48	4.768	9
1133	8999.5	9.6	523.56	1.74	0.18125	48	4.777	8
1132	9009.1	9.6	525.30	1.75	0.18173	48	4.785	9
1131	9018.7	9.6	527.05	1.76	0.18221	48	4.794	8

TABLE I—Continued.

V	S_v	*Diff.*	A_v	*Diff.*	I_v	*Diff.*	T_v	*Diff.*
1130	9028.3	9.7	528.81	1.77	0.18269	49	4.802	9
1129	9038.0	9.8	530.58	1.78	0.18318	49	4.811	9
1128	9047.8	9.7	532.36	1.79	0.18367	49	4.820	8
1127	9057.5	9.8	534.15	1.80	0.18416	50	4.828	9
1126	9067.3	9.9	535.95	1.82	0.18466	50	4.837	9
1125	9077.2	9.9	537.77	1.83	0.18516	50	4.846	8
1124	9087.1	9.9	539.60	1.85	0.18566	51	4.854	9
1123	9097.0	9.9	541.45	1.86	0.18617	51	4.863	8
1122	9106.9	10.0	543.31	1.87	0.18668	51	4.871	9
1121	9116.9	10.0	545.18	1.88	0.18719	51	4.880	9
1120	9126.9	10.1	547.06	1.89	0.18770	52	4.889	9
1119	9137.0	10.1	548.95	1.90	0.18822	52	4.898	9
1118	9147.1	10.1	550.85	1.91	0.18874	52	4.907	9
1117	9157.2	10.1	552.76	1.92	0.18926	52	4.916	10
1116	9167.3	10.2	554.68	1.94	0.18978	53	4.926	9
1115	9177.5	10.2	556.62	1.95	0.19031	53	4.935	9
1114	9187.7	10.3	558.57	1.96	0.19084	53	4.944	9
1113	9198.0	10.3	560.53	1.97	0.19137	53	4.953	10
1112	9208.3	10.4	562.50	1.99	0.19190	54	4.963	9
1111	9218.7	10.4	564.49	2.01	0.19244	54	4.972	9
1110	9229.1	10.4	566.50	2.01	0.19298	54	4 981	10
1109	9239.5	10.4	568.51	2.02	0.19352	55	4.991	9
1108	9249.9	10.5	570.53	2.03	0.19407	55	5.000	10
1107	9260.4	10.5	572.56	2.05	0.19462	55	5.010	9
1106	9270.9	10.6	574.61	2.06	0.19517	56	5.019	10
1105	9281.5	10.6	576.67	2.08	0.19573	56	5.029	9
1104	9292.1	10.6	578.75	2.09	0.19629	56	5.038	10
1103	9302.7	10.7	580.84	2.11	0.19685	57	5.048	9
1102	9313.4	10.7	582.95	2.13	0.19742	57	5.057	10
1101	9324.1	10.8	585.08	2.15	0.19799	57	5.067	10
1100	9334.9	10.8	587.23	2.16	0.19856	58	5.077	10
1099	9345.7	10.9	589.39	2.17	0.19914	58	5.087	10
1098	9356.6	10.9	591.56	2.18	0.19972	58	5.097	10
1097	9367.5	10.9	593.74	2.19	0.20030	58	5.107	10
1096	9378.4	10.9	595.93	2.21	0.20088	59	5.117	10
1095	9389.3	11.0	598.14	2.22	0.20147	59	5.127	10
1094	9400.3	11.0	600.36	2.23	0.20206	59	5.137	10
1093	9411.3	11.1	602.59	2.24	0.20265	60	5.147	10
1092	9422.4	11.1	604.83	2.25	0.20325	60	5.157	10
1091	9433.5	11.2	607.08	2.27	0.20385	60	5.167	10

TABLE I—CONTINUED.

V	S_v	*Diff.*	A_v	*Diff.*	I_v	*Diff.*	T_v	*Diff.*
1090	9444.7	11.2	609.35	2.29	0.20445	61	5.177	10
1089	9455.9	11.2	611.64	2.31	0.20506	61	5.187	10
1088	9467.1	11.3	613.95	2.33	0.20567	61	5.197	10
1087	9478.4	11.3	616.28	2.35	0.20628	62	5.207	10
1086	9489.7	11.4	618.63	2.36	0.20690	62	5.217	10
1085	9501.1	11.4	620.99	2.37	0.20752	62	5.227	11
1084	9512.5	11.5	623.36	2.38	0.20814	63	5.238	11
1083	9524.0	11.5	625.74	2.40	0.20877	63	5.249	11
1082	9535.5	11.5	628.14	2.41	0.20940	63	5.260	11
1081	9547.0	11.6	630.55	2.43	0.21003	64	5.271	11
1080	9558.6	11.6	632.98	2.45	0.21067	64	5.282	11
1079	9570.2	11.7	635.43	2.47	0.21131	65	5.293	11
1078	9581.9	11.7	637.90	2.48	0.21196	65	5.304	11
1077	9593.6	11.7	640.38	2.50	0.21261	65	5.315	11
1076	9605.3	11.8	642.88	2.52	0.21326	66	5.326	11
1075	9617.1	11.8	645.40	2.54	0.21392	66	5.337	11
1074	9628.9	11.9	647.94	2.56	0.21458	66	5.348	11
1073	9640.8	12.0	650.50	2.57	0.21524	67	5.359	11
1072	9652.8	12.0	653.07	2.59	0.21591	67	5.370	11
1071	9664.8	12.0	655.66	2.61	0.21658	67	5.381	11
1070	9676.8	12.0	658.27	2.63	0.21725	68	5.392	11
1069	9688.8	12.1	660.90	2.64	0.21793	68	5.403	11
1068	9700.9	12.2	663.54	2.66	0.21861	69	5.414	11
1067	9713.1	12.2	666.20	2.68	0.21930	69	5.425	11
1066	9725.3	12.2	668.88	2.70	0.21999	69	5.436	11
1065	9737.5	12.3	671.58	2.72	0.22068	70	5.447	12
1064	9749.8	12.3	674.30	2.74	0.22138	70	5.459	12
1063	9762.1	12.4	677.04	2.75	0.22208	71	5.471	12
1062	9774.5	12.5	679.79	2.77	0.22279	71	5.483	12
1061	9786.0	12.5	682.56	2.79	0.22350	71	5.495	12
1060	9798.5	12.6	685.35	2.81	0.22421	72	5.507	12
1059	9811.1	12.6	688.16	2.83	0.22493	72	5.519	12
1058	9823.7	12.7	690.99	2.85	0.22565	72	5.531	12
1057	9836.4	12.8	693.84	2.87	0.22637	72	5.543	12
1056	9849.2	12.8	696.71	2.89	0.22710	73	5.555	13
1055	9862.0	12.9	699.60	2.91	0.22783	74	5.568	12
1054	6874.9	12.9	702.51	2.93	0.22857	74	5.580	12
1053	9887.8	13.0	705.44	2.96	0.22931	75	5.592	12
1052	9900.8	13.0	708.40	2.98	0.23006	75	5.604	12
1051	9913.8	13.0	711.38	3.00	0.23081	76	5.616	12

TABLE I—CONTINUED.

V	S_v	*Diff.*	A_v	*Diff.*	I_v	*Diff.*	T_v	*Diff.*
1050	9926.8	13.0	714.38	3.02	0.23157	76	5.628	12
1049	9939.8	13.1	717.40	3.04	0.23233	76	5.640	12
1048	9952.9	13.1	720.44	3.06	0.23309	77	5.652	12
1047	9966.0	13.1	723.50	3.08	0.23386	77	5.664	13
1046	9979.1	13.3	726.58	3.11	0.23463	78	5.677	13
1045	9992.4	13.3	729.69	3.13	0.23541	78	5.690	13
1044	10005.7	13.3	732.82	3.15	0.23619	79	5.703	13
1043	10019.0	13.3	735.97	3.17	0.23698	79	5.716	13
1042	10032.3	13.4	739.14	3.19	0.23777	80	5.729	13
1041	10045.7	13.5	742.33	3.21	0.23857	80	5.742	13
1040	10059.2	13.5	745.54	3.24	0.23937	81	5.755	13
1039	10072.7	13.6	748.78	3.27	0.24018	81	5.768	13
1038	10086.3	13.6	752.05	3.29	0.24099	81	5.781	13
1037	10099.9	13.7	755.34	3.32	0.24180	82	5.794	13
1036	10113.6	13.7	758.66	3.35	0.24262	82	5.807	13
1035	10127.3	13.8	762.01	3.37	0.24344	83	5.820	13
1034	10141.1	13.8	765.38	3.40	0.24427	83	5.833	13
1033	10154.9	13.8	768.78	3.42	0.24510	84	5.846	13
1032	10168.7	13.9	772.20	3.45	0.24594	84	5.859	14
1031	10182.6	14.1	775.65	3.48	0.24678	85	5.873	14
1030	10196.7	14.1	779.13	3.50	0.24763	85	5.887	14
1029	10210.8	14.1	782.63	3.52	0.24848	86	5.901	14
1028	10224.9	14.1	786.15	3.54	0.24934	86	5.915	14
1027	10239.0	14.2	789.69	3.56	0.25020	87	5.929	14
1026	10253.2	14.3	793.25	3.58	0.25107	87	5.943	14
1025	10267.5	14.3	796.83	3.60	0.25194	88	5.957	14
1024	10281.8	14.4	800.43	3.63	0.25282	88	5.971	14
1023	10296.2	14.5	804.06	3.65	0.25370	89	5.985	14
1022	10310.7	14.5	807.71	3.67	0.25459	89	5.999	14
1021	10325.2	14.5	811.38	3.69	0.25548	90	6.013	14
1020	10339.7	14.6	815.07	3.72	0.25638	90	6.027	14
1019	10354.3	14.7	818.79	3.76	0.25728	91	6.041	14
1018	10369.0	14.7	822.55	3.79	0.25819	92	6.055	14
1017	10383.7	14.8	826.34	3.82	0.25911	92	6.069	14
1016	10398.5	14.8	830.16	3.86	0.26003	93	6.083	15
1015	10413.3	14.9	834.02	3.90	0.26096	93	6.098	15
1014	10428.2	15.0	837.92	3.93	0.26189	94	6.113	15
1013	10443.2	15.0	841.85	3.97	0.26283	94	6.128	15
1012	10458.2	15.1	845.82	4.01	0.26377	95	6.143	15
1011	10473.3	15.1	849.83	4.05	0.26472	95	6.158	15

TABLE I—CONTINUED.

V	S_v	*Diff.*	A_v	*Diff.*	I_v	*Diff.*	$\bar{T}_v$	*Diff.*
1010	10488.4	15.2	853.88	4.07	0.26567	96	6.173	15
1009	10503.6	15.3	857.95	4.09	0.26663	96	6.188	15
1008	10518.9	15.3	862.04	4.11	0.26759	97	6.203	15
1007	10534.2	15.4	866.15	4.14	0.26856	98	6.218	15
1006	10549.6	15.4	870.29	4.17	0.26954	98	6.233	15
1005	10565.0	15.5	874.46	4.20	0.27052	99	6.248	15
1004	10580.5	15.6	878.66	4.23	0.27151	100	6.263	16
1003	10596.1	15.6	882.89	4.26	0.27251	100	6.279	16
1002	10611.7	15.7	887.15	4.28	0.27351	101	6.295	16
1001	10627.4	15.7	891.43	4.31	0.27452	101	6.311	16
1000	10643.1	15.8	895.74	4.35	0.27553	102	6.327	16
999	10658.9	15.9	900.09	4.38	0.27655	102	6.343	16
998	10674.8	16.0	904.47	4.42	0.27757	103	6.359	16
997	10690.8	16.0	908.89	4.46	0.27860	103	6.375	16
996	10706.8	16.1	913.35	4.50	0.27963	104	6.391	16
995	10722.9	16.1	917.85	4.54	0.28067	105	6.407	16
994	10739.0	16.2	922.39	4.58	0.28172	106	6.423	16
993	10755.2	16.2	926.97	4.62	0.28278	106	6.439	16
992	10771.4	16.3	931.59	4.66	0.28384	107	6.455	17
991	10787.7	16.4	936.25	4.70	0.28491	107	6.472	17
990	10804.1	16.5	940.95	4.73	0.28598	108	6.489	17
989	10820.6	16.6	945.68	4.76	0.28706	108	6.506	17
988	10837.2	16.6	950.44	4.79	0.28814	110	6.523	17
987	10853.8	16.6	955.23	4.83	0.28924	110	6.540	17
986	10870.4	16.7	960.06	4.87	0.29034	111	6.557	17
985	10887.1	16.8	964.93	4.90	0.29145	111	6.574	17
984	10903.9	16.9	969.83	4.94	0.29256	112	6.591	17
983	10920.8	16.9	974.77	4.98	0.29368	113	6.608	17
982	10937.7	17.0	979.75	5.02	0.29481	114	6.625	17
981	10954.7	17.1	984.77	5.06	0.29595	115	6.642	17
980	10971.8	17.2	989.83	5.10	0.29710	115	6.659	18
979	10989.0	17.2	994.93	5.14	0.29825	116	6.677	18
978	11006.2	17.3	1000.07	5.18	0.29941	116	6.695	18
977	11023.5	17.3	1005.25	5.23	0.30057	117	6.713	18
976	11040.8	17.5	1010.48	5.27	0.30174	118	6.731	18
975	11058.3	17.5	1015.75	5.32	0.30292	119	6.749	18
974	11075.8	17.6	1021.07	5.36	0.30411	119	6.767	18
973	11093.4	17.6	1026.43	5.40	0.30530	120	6.785	18
972	11111.0	17.7	1031.83	5.44	0.30650	121	6.803	18
971	11128.7	17.8	1037.27	5.48	0.30771	121	6.821	18

TABLE I—Continued.

V	S_v	*Diff.*	A_v	*Diff.*	I_v	*Diff.*	T_v	*Diff.*
970	11146.5	17.9	1042.75	5.52	0.30892	122	6.839	18
969	11164.4	18.0	1048.27	5.56	0.31014	123	6.857	18
968	11182.4	18.0	1053.83	5.60	0.31137	124	6.875	18
967	11200.4	18.0	1059.43	5.64	0.31261	124	6.893	19
966	11218.4	18.1	1065.07	5.68	0.31385	125	6.912	19
965	11236.5	18.1	1070.75	5.72	0.31510	125	6.931	19
964	11254.6	18.2	1076.47	5.76	0.31635	126	6.950	19
963	11272.8	18.2	1082.23	5.80	0.31761	126	6.969	19
962	11291.0	18.2	1088.03	5.85	0.31887	127	6.988	19
961	11309.3	18.3	1093.88	5.89	0.32014	128	7.007	19
960	11327.5	18.3	1099.77	5.91	0.32142	128	7.026	19
959	11345.8	18.3	1105.68	5.95	0.32270	128	7.045	19
958	11364.1	18.4	1111.63	5.97	0.32498	129	7.064	19
957	11382.5	18.4	1117.60	6.01	0.32527	129	7.083	19
956	11400.9	18.4	1123.61	6.05	0.32656	130	7.102	19
955	11419.3	18.5	1129.66	6.09	0.32786	130	7.121	19
954	11437.8	18.6	1135.75	6.12	0.32916	131	7.140	20
953	11456.4	18.6	1141.87	6.15	0.33047	132	7.160	20
952	11475.0	18.6	1148.02	6.18	0.33179	133	7.180	20
951	11493.6	18.6	1154.20	6.22	9.33312	133	7.200	20
950	11511.2	18.7	1160.42	6.26	0.33445	134	7.220	19
949	11529.9	18.7	1166.68	6.29	0.33579	134	7.239	20
948	11548.6	18.8	1172.97	6.33	0.33713	134	7.259	20
947	11567.4	18.8	1179.30	6.37	0.33847	135	7.279	20
946	11586.2	18.8	1185.67	6.41	0.33982	135	7.299	20
945	11605.0	18.9	1192.08	6.45	0.34117	136	7.319	20
944	11623.9	18.9	1198.53	6.49	0.34253	137	7.339	20
943	11642.8	19.0	1205.02	6.53	0.34390	137	7.359	20
942	11661.8	19.0	1211.55	6.57	0.34527	138	7.379	20
941	11680.8	19.1	1218.12	6.61	0.34665	139	7.399	20
940	11700.9	19.1	1224.73	6.65	0.34804	139	7.419	20
939	11720.0	19.1	1231.38	6.70	0.34943	140	7.439	20
938	11739.1	19.2	1238.08	6.74	0.35083	140	7.459	20
937	11758.3	19.2	1244.82	6.78	0.35223	141	7.479	20
936	11777.5	19.2	1251.60	6.82	0.35364	141	7.499	21
935	11796.7	19.3	1258.42	6.87	0.35505	142	7.520	21
934	11816.0	19.3	1265.29	6.91	0.35647	143	7.541	21
933	11835.3	19.4	1272.20	6.95	0.35790	143	7.562	21
932	11854.7	19.4	1279.15	6.99	0.35933	144	7.583	21
931	11874.1	19.4	1286.14	7.03	0.36077	145	7.604	21

TABLE I—CONTINUED.

V	S_v	*Diff.*	A_v	*Diff.*	I_v	*Diff.*	T_v	*Diff.*
930	11893.5	19.5	1293.17	7.09	0.36222	145	7.625	21
929	11913.0	19.6	1300.26	7.13	0.36367	146	7.646	21
928	11932.6	19.6	1307.39	7.18	0.36513	146	7.667	21
927	11952.2	19.7	1314.57	7.21	0.36659	147	7.688	21
926	11971.9	19.7	1321.78	7.26	0.36806	148	7.709	21
925	11991.6	19.7	1329.04	7.30	0.36954	148	7.730	21
924	12011.3	19.7	1336.34	7.34	0.37102	149	7.751	21
923	12031.0	19.8	1343.68	7.38	0.37251	150	7.772	22
922	12050.8	19.8	1351.06	7.42	0.37401	150	7.794	22
921	12070.6	19.9	1358.48	7.47	0.37551	151	7.816	22
920	12090.5	20.0	1365.95	7.52	0.37702	152	7.838	22
919	12110.5	20.0	1373.47	7.57	0.37854	152	7.860	22
918	12130.5	20.0	1381.04	7.62	0.38006	153	7.882	22
917	12150.5	20.0	1388.66	7.67	0.38159	154	7.904	22
916	12170.5	20.1	1396.33	7.72	0.38313	154	7.926	22
915	12190.6	20.1	1404.05	7.77	0.38467	155	7.948	22
914	12210.7	20.2	1411.82	7.82	0.38622	156	7.970	22
913	12230.9	20.2	1419.64	7.87	0.38778	156	7.992	22
912	12251.1	20.3	1427.51	7.91	0.38934	157	8.014	22
911	12271.4	20.4	1435.42	7.96	0.39091	158	8.036	22
910	12291.8	20.4	1443.38	8.01	0.39249	158	8.058	22
909	12312.2	20.4	1451.39	8.06	0.39407	159	8.080	22
908	12332.6	20.4	1459.45	8.11	0.39566	160	8.102	22
907	12353.0	20.5	1467.56	8.17	0.39726	160	8.124	23
906	12373.5	20.5	1475.73	8.22	0.39886	161	8.147	23
905	12394.0	20.6	1483.95	8.27	0.40047	162	8.170	23
904	12414.6	20.7	1492.22	8.32	0.40209	163	8.193	23
903	12435.3	20.7	1500.54	8.37	0.40372	163	8.216	23
902	12456.0	20.7	1508.91	8.41	0.40535	164	8.239	23
901	12476.7	20.8	1517.32	8.46	0.40699	165	8.262	23
900	12497.5	20.8	1525.78	8.52	0.40864	166	8.285	23
899	12518.3	20.9	1534.30	8.58	0.41030	166	8.308	23
898	12539.2	20.9	1542.88	8.63	0.41196	167	8.331	23
897	12560.1	21.0	1551.51	8.69	0.41363	168	8.354	23
896	12581.1	21.0	1560.20	8.75	0.41531	168	8.377	23
895	12602.1	21.1	1568.95	8.81	0.41699	169	8.400	23
894	12623.2	21.1	1577.76	8.86	0.41868	170	8.423	23
893	12644.3	21.1	1586.62	8.92	0.42038	171	8.446	24
892	12665.4	21.2	1595.54	8.96	0.42209	172	8.470	24
891	12686.6	21.3	1604.50	9.01	0.42381	172	8.494	24

TABLE I—Continued.

V	S_v	*Diff.*	A_v	*Diff.*	I_v	*Diff.*	T_v	*Diff.*
890	12707.9	21.3	1613.51	9.07	0.42553	173	8.518	24
889	12729.2	21.3	1622.58	9.13	0.42726	174	8.542	24
888	12750.5	21.4	1631.71	9.19	0.42900	175	8.566	24
887	12771.9	21.4	1640.90	9.25	0.43075	176	8.590	24
886	12793.3	21.5	1650.15	9.31	0.43251	176	8.614	24
885	12814.8	21.6	1659.46	9.37	0.43427	177	8.638	24
884	12836.4	21.6	1668.83	9.44	0.43604	178	8.662	25
883	12858.0	21.6	1678.27	9.50	0.43782	179	8.687	25
882	12879.6	21.7	1687.77	9.56	0.43961	179	8.712	25
881	12901.3	21.7	1697.33	9.62	0.44140	180	8.737	25
880	12923.0	21.8	1706.95	9.68	0.44320	181	8.762	25
879	12944.8	21.8	1716.63	9.74	0.44501	182	8.787	25
878	12966.6	21.9	1726.37	9.80	0.44683	183	8.812	25
877	12988.5	21.9	1736.17	9.87	0.44866	184	8.837	25
876	13010.4	22.0	1746.04	9.93	0.45050	184	8.862	25
875	13032.4	22.0	1755.97	9.99	0.45234	185	8.887	25
874	13054.4	22.1	1765.96	10.05	0.45419	186	8.912	25
873	13076.5	22.2	1776.01	10.11	0.45605	187	8.937	26
872	13098.7	22.2	1786.12	10.17	0.45792	188	8.963	26
871	13120.9	22.2	1796.29	10.23	0.45980	189	8.989	26
870	13143.1	22.3	1806.52	10.30	0.46169	190	9.015	26
869	13165.4	22.3	1816.82	10.37	0.46359	191	9.041	26
868	13187.7	22.4	1827.19	10.44	0.46550	191	9.067	26
867	13210.1	22.4	1837.63	10.51	0.46741	192	9.093	26
866	13232.5	22.5	1848.14	10.58	0.46933	193	9.119	26
865	13255.0	22.6	1858.72	10.65	0.47126	194	9.145	26
864	13277.6	22.6	1869.37	10.72	0.47320	195	9.171	26
863	13300.2	22.6	1881.09	10.79	0.47515	196	9.197	26
862	13322.8	22.7	1891.88	10.86	0.47711	197	9.223	26
861	13345.5	22.8	1902.74	10.92	0.47908	197	9.249	26
860	13368.3	22.8	1912.66	11.00	0.48105	198	9.275	27
859	13391.1	22.8	1923.66	11.07	0.48303	199	9.302	27
858	13413.9	22.9	1934.73	11.14	0.48502	201	9.329	27
857	13436.8	23.0	1945.87	11.21	0.48703	202	9.356	27
856	13459.8	23.0	1957.08	11.29	0.48905	202	9.383	27
855	13482.8	23.1	1968.37	11.36	0.49107	204	9.410	27
854	13505.9	23.1	1979.73	11.43	0.49311	204	9.437	27
853	13529.0	23.2	1991.16	11.50	0.49515	205	9.464	27
852	13552.2	23.2	2002.66	11.57	0.49720	206	9.491	27
851	13575.4	23.3	2014.23	11.64	0.49926	207	9.518	27

TABLE I—Continued.

V	S_v	*Diff.*	A_v	*Diff.*	I_v	*Diff.*	T_v	*Diff.*
850	13598.7	23.4	2025.87	11.72	0.50133	208	9.545	27
849	13622.1	23.4	2037.59	11.80	0.50341	210	9.572	28
848	13645.5	23.5	2049.39	11.88	0.50551	210	9.600	28
847	13669.0	23.5	2061.27	11.96	0.50761	211	9.628	28
846	13692.5	23.6	2073.23	12.04	0.50972	212	9.656	28
845	13716.1	23.6	2085.27	12.12	0.51184	213	9.684	28
844	13739.7	23.7	2097.39	12.20	0.51397	214	9.712	28
843	13763.4	23.7	2109.59	12.28	0.51611	215	9.740	28
842	13787.1	23.8	2121.87	12.36	0.51826	216	9.768	28
841	13810.9	23.8	2134.23	12.45	0.52042	217	9.796	28
840	13834.7	23.9	2146.68	12.53	0.52259	218	9.824	29
839	13858.6	24.0	2159.21	12.61	0.52477	219	9.853	29
838	13882.6	24.0	2171.82	12.69	0.52696	220	9.882	29
837	13906.6	24.1	2184.51	12.78	0.52916	222	9.911	29
836	13930.7	24.2	2197.29	12.86	0.53138	223	9.940	29
835	13954.9	24.2	2210.15	12.94	0.53361	224	9.969	29
834	13979.1	24.2	2223.09	13.02	0.53585	225	9.998	29
833	14003.3	24.3	2236.11	13.10	0.53810	226	10.027	29
832	14027.6	24.4	2249.21	13.18	0.54036	226	10.056	29
831	14052.0	24.4	2262.39	13.26	0.54262	227	10.085	29
830	14076.4	24.5	2275.65	13.36	0.54489	229	10.114	30
829	14100.9	24.6	2289.01	13.45	0.54718	230	10.144	30
828	14125.5	24.6	2302.46	13.55	0.54948	232	10.174	30
827	14150.1	24.6	2316.01	13.64	0.55180	232	10.204	30
826	14174.7	24.7	2329.65	13.73	0.55412	233	10.234	30
825	14199.4	24.8	2343.38	13.83	0.55645	234	10.264	30
824	14224.2	24.9	2357.21	13.92	0.55879	236	10.294	30
823	14249.1	24.9	2371.13	14.01	0.56115	237	10.324	30
822	14274.0	24.9	2385.14	14.10	0.56352	238	10.354	30
821	14298.9	25.0	2399.24	14.19	0.56590	239	10.384	30
820	14323.9	25.1	2413.43	14.28	0.56829	240	10.414	31
819	14349.0	25.2	2427.71	14.38	0.57069	241	10.445	31
818	14374.2	25.2	2442.09	14.48	0.57310	243	10.476	31
817	14399.4	25.3	2456.57	14.58	0.57553	244	10.507	31
816	14424.7	25.3	2471.15	14.68	0.57797	245	10.538	31
815	14450.0	25.4	2485.83	14.78	0.58042	246	10.569	31
814	14475.4	25.5	2500.61	14.88	0.58288	247	10.600	31
813	14500.9	25.5	2515.49	14.97	0.58535	249	10.631	31
812	14526.4	25.6	2530.46	15.07	0.58784	250	10.662	31
811	14552.0	25.7	2545.53	15.17	0.59034	251	10.693	32

TABLE I—CONTINUED.

V	S_v	*Diff.*	A_v	*Diff.*	I_v	*Diff.*	T_v	*Diff.*
810	14577.7	25.7	2560.70	15.27	0.59285	252	10.725	32
809	14603.4	25.8	2575.97	15.38	0.59537	254	10.757	32
808	14629.2	25.8	2591.35	15.49	0.59791	255	10.789	32
807	14655.0	25.9	2606.84	15.60	0.60046	256	10.821	32
806	14680.9	26.0	2622.44	15.71	0.60302	258	10.853	32
805	14706.9	26.0	2638.15	15.81	0.60560	259	10.885	32
804	14732.9	26.1	2653.96	15.91	0.60819	260	10.917	32
803	14759.0	26.2	2669.87	16.01	0.61079	262	10.949	33
802	14785.2	26.2	2685.88	16.11	0.61341	262	10.982	33
801	14811.4	26.3	2701.99	16.21	0.61603	263	11.015	33
800	14837.7	26.3	2718.20	16.32	0.61866	265	11.048	33
799	14864.0	26.4	2734.52	16.44	0.62131	267	11.081	33
798	14890.4	26.4	2750.96	16.55	0.62398	269	11.114	33
797	14916.8	26.5	2767.51	16.67	0.62667	270	11.147	33
796	14943.3	26.6	2784.18	16.79	0.62937	272	11.180	33
795	14969.9	26.7	2800.97	16.91	0.63209	273	11.213	34
794	14996.6	26.8	2817.88	17.03	0.63482	273	11.247	34
793	15023.4	26.9	2834.91	17.15	0.63755	274	11.281	34
792	15050.3	27.0	2852.06	17.29	0.64029	275	11.315	34
791	15077.3	27.1	2869.35	17.39	0.64304	276	11.349	34
790	15104.4	27.1	2886.74	17.51	0.64580	278	11.383	34
789	15131.5	27.1	2904.25	17.62	0.64858	280	11.417	34
788	15158.6	27.2	2921.87	17.74	0.65138	282	11.451	35
787	15185.8	27.2	2939.61	17.85	0.65420	283	11.486	35
786	15213.0	27.2	2957.46	17.97	0.65703	285	11.521	35
785	15240.2	27.3	2975.43	18.08	0.65988	286	11.556	35
784	15267.5	27.3	2993.51	18.20	0.66274	286	11.591	35
783	15294.8	27.3	3011.71	18.31	0.66560	287	11.626	35
782	15322.1	27.3	3030.02	18.42	0.66847	288	11.661	35
781	15349.4	27.4	3048.44	18.53	0.67135	289	11.696	35
780	15376.8	27.5	3066.97	18.62	0.67424	290	11.731	35
779	15404.3	27.5	3085.59	18.72	0.67714	292	11.766	35
778	15431.8	27.5	3104.31	18.82	0.68006	293	11.801	35
777	15459.3	27.6	3123.13	18.91	0.68299	294	11.836	35
776	15486.9	27.6	3142.04	19.00	0.68593	294	11.871	36
775	15514.5	27.6	3161.04	19.10	0.68887	295	11.907	36
774	15542.1	27.6	3180.14	19.19	0.69182	297	11.943	36
773	15569.7	27.7	3199.33	19.30	0.69479	298	11.979	36
772	15597.4	27.7	3218.63	19.39	0.69777	300	12.015	36
771	15625.1	27.7	3238.02	19.50	0.70077	302	12.051	36

TABLE I—CONTINUED.

V	S_v	*Diff.*	A_v	*Diff.*	I_v	*Diff.*	T_v	*Diff.*
770	15652.8	27.8	3257.52	19.61	0.70379	303	12.087	36
769	15680.6	27.8	3277.13	19.73	0.70682	304	12.123	36
768	15708.4	27.8	3296.86	19.85	0.70986	304	12.159	36
767	15736.2	27.9	3316.71	19.97	0.71290	305	12.195	36
766	15764.1	28.0	3336.68	20.09	0.71595	306	12.231	36
765	15792.1	28.0	3356.77	20.21	0.71901	308	12.267	37
764	15820.1	28.0	3376.98	20.32	0.72209	309	12.304	37
763	15848.1	28.0	3397.30	20.43	0.72518	310	12.341	37
762	15876.1	28.0	3417.73	20.54	0.72828	311	12.378	37
761	15904.1	28.1	3438.27	20.65	0.73139	313	12.415	37
760	15932.2	28.2	3458.92	20.76	0.73452	314	12.452	37
759	15960.4	28.2	3479.68	20.88	0.73766	315	12.489	37
758	15988.6	28.2	3500.56	21.00	0.74081	316	12.526	37
757	16016.8	28.3	3521.56	21.11	0.74397	318	12.563	37
756	16045.1	28.3	3542.67	21.23	0.74715	319	12.600	37
755	16073.4	28.3	3563.90	21.36	0.75034	320	12.637	38
754	16101.7	28.4	3585.26	21.48	0.75354	322	12.675	38
753	16130.1	28.4	3606.74	21.60	0.75676	323	12.713	38
752	16158.5	28.5	3628.34	21.72	0.75999	324	12.751	38
751	16186.0	28.5	3650.06	21.84	0.76323	325	12.789	38
750	16215.5	28.5	3671.90	21.95	0.76648	326	12.827	38
749	16244.0	28.6	3693.85	22.08	0.76974	328	12.865	38
748	16272.6	28.6	3715.93	22.20	0.77302	329	12.903	38
747	16301.2	28.7	3738.13	22.33	0.77631	331	12.941	38
746	16329.9	28.7	3760.46	22.46	0.77962	333	12.979	39
745	16358.6	28.7	3782.92	22.59	0.78295	334	13.018	39
744	16387.3	28.8	3805.51	22.72	0.78629	335	13.057	39
743	16416.1	28.8	3828.23	22.84	0.78964	336	13.096	39
442	16444.9	28.9	3851.07	22.96	0.79300	337	13.135	39
741	16473.8	28.9	3874.03	23.09	0.79637	338	13.174	39
740	16502.7	28.9	3897.12	23.21	0.79975	339	13.213	39
739	16531.6	29.0	3920.33	23.34	0.80314	341	13.252	39
738	16560.6	29.0	3943.67	23.48	0.80655	343	13.291	39
737	16589.6	29.0	3967.15	23.62	0.80998	345	13.330	39
736	16618.6	29.1	3990.77	23.75	0.81343	347	13.369	40
735	16647.7	29.1	4014.52	23.89	0.81690	348	13.409	40
734	16676.8	29.1	4038.41	24.03	0.82038	349	13.449	40
733	16705.9	29.2	4062.44	24.16	0.82387	350	13.489	40
732	16735.1	29.2	4086.60	24.29	0.82737	351	13.529	40
731	16764.3	29.3	4110.89	24.42	0.83088	352	13.569	40

TABLE I—CONTINUED.

V	S_v	*Diff.*	A_v	*Diff.*	I_v	*Diff.*	T_v	*Diff.*
730	16793.6	29.3	4135.31	24.56	0.83440	354	13.609	40
729	16822.9	29.4	4159.87	24.70	0.83794	356	13.649	40
728	16852.3	29.4	4184.57	24.83	0.84150	358	13.689	40
727	16881.7	29.4	4209.40	24.97	0.84508	359	13.729	40
726	16911.1	29.5	4234.37	25.11	0.84867	361	13.769	41
725	16940.6	29.5	4259.48	25.25	0.85228	362	13.810	41
724	16970.1	29.6	4284.73	25.39	0.85590	363	13.851	41
723	16999.7	29.6	4310.12	25.53	0.85953	364	13.892	41
722	17029.3	29.6	4335.65	25.67	0.86327	366	13.933	41
721	17058.9	29.6	4361.32	25.81	0.86683	367	13.974	41
720	17088.5	29.7	4387.13	25.96	0.87050	369	14.015	41
719	17118.2	29.8	4413.09	26.13	0.87419	371	14.056	42
718	17148.0	29.8	4439.22	26.28	0.87790	372	14.098	42
717	17177.8	29.9	4465.50	26.43	0.88162	374	14.140	42
716	17207.7	29.9	4491.93	26.58	0.88546	376	14.182	42
715	17237.6	29.9	4518.51	26.74	0.88922	377	14.224	42
714	17267.5	30.0	4545.25	26.89	0.89299	378	14.260	42
713	17297.5	30.0	4572.14	27.04	0.89677	380	14.308	42
712	17327.5	30.1	4599.18	27.20	0.90057	382	14.350	42
711	17357.6	30.1	4526.38	27.36	0.90439	384	14.392	42
710	17387.7	30.1	4653.74	27.50	0.90813	386	14.434	43
709	17417.8	30.2	4681.24	27.65	0.91199	387	14.477	43
708	17448.0	30.3	4708.89	27.81	0.91586	388	14.520	43
707	17478.3	30.3	4736.70	27.96	0.91974	390	14.563	43
706	17508.6	30.3	4764.66	28.12	0.92364	392	14.606	43
705	17538.9	30.3	4792.78	28.27	0.92756	394	14.649	43
704	17569.2	30.4	4821.05	28.42	0.93150	395	14.692	43
703	17599.6	30.4	4849.47	28.57	0.93545	396	14.735	43
702	17630.0	30.5	4878.04	28.73	0.93941	398	14.778	43
701	17660.5	30.5	4906.77	28.88	0.94339	400	14.821	43
700	17691.0	30.6	4935.65	29.05	0.94739	402	14.864	44
699	17721.6	30.7	4964.70	29.22	0.95141	404	14.908	44
698	17752.3	30.7	4993.92	29.40	0.95545	405	14.952	44
697	17783.0	30.7	5023.32	29.57	0.95950	407	14.996	44
696	17813.7	30.7	5052.89	29.74	0.96357	409	15.040	44
695	17844.4	30.7	5082.63	29.92	0.96766	411	15.084	44
694	17875.1	30.8	5112.55	30.10	0.97177	412	15.128	44
693	17905.9	30.9	5142.65	30.28	0.97589	414	15.172	45
692	17936.8	31.0	5172.93	30.45	0.98003	416	15.217	45
691	17967.8	31.0	5203.38	30.63	0.98419	418	15.262	45

TABLE I—CONTINUED.

V	S_v	*Diff.*	A_v	*Diff.*	I_v	*Diff.*	T_v	*Diff.*
690	17998.8	31.0	5234.01	30.80	0.98837	420	15.307	45
689	18029.8	31.1	5264.81	30.97	0.99257	421	15.352	45
688	18060.9	31.1	5295.78	31.14	0.99678	423	15.397	45
687	18092.0	31.1	5326.92	31.32	1.00101	425	15.442	45
686	18123.1	31.2	5358.24	31.49	1.00526	427	15.487	46
685	18154.3	31.2	5389.73	31.66	1.00953	429	15.533	46
684	18185.5	31.3	5421.39	31.83	1.01382	431	15.579	46
683	18216.8	31.3	5453.22	32.00	1.01813	433	15.625	46
682	18248.1	31.4	5485.22	32.17	1.02246	435	15.671	46
681	18279.5	31.4	5517.39	32.34	1.02681	436	15.717	46
680	18310.9	31.5	5549.73	32.54	1.03117	438	15.763	46
679	18342.4	31.5	5582.27	32.75	1.03555	440	15.809	47
678	18373.9	31.6	5615.02	32.96	1.03995	442	15.856	47
677	18405.5	31.6	5647.98	33.17	1.04437	445	15.903	47
676	18437.1	31.7	5681.15	33.38	1.04882	447	15.950	47
675	18468.8	31.7	5714.53	33.59	1.05329	449	15.997	47
674	18500.5	31.7	5748.12	33.79	1.05778	450	16.044	47
673	18532.2	31.8	5781.91	34.00	1.06228	452	16.091	47
672	18564.0	31.9	5815.91	34.21	1.06680	454	16.138	47
671	18595.9	31.9	5850.12	34.42	1.07134	456	16.185	47
670	18627.8	31.9	5884.54	34.60	1.07590	458	16.232	48
669	18659.7	32.0	5919.14	34.78	1.08048	460	16.280	48
668	18691.7	32.0	5953.92	34.96	1.08508	463	16.328	48
667	18723.7	32.1	5988.88	35.14	1.08971	465	16.376	48
666	18755.8	32.2	6024.02	35.32	1.09436	467	16.424	48
665	18788.0	32.2	6059.34	35.51	1.09903	469	16.472	48
664	18820.2	32.2	6094.85	35.69	1.10372	471	16.520	49
663	18852.4	32.2	6130.54	35.86	1.10843	473	16.569	49
662	18884.6	32.3	6166.40	36.04	1.11316	475	16.618	49
661	18916.9	32.4	6202.44	36.22	1.11791	477	16.667	49
660	18949.3	32.4	6238.66	36.45	1.12268	479	16.716	49
659	18981.7	32.5	6275.11	36.68	1.12747	482	16.765	49
658	19014.2	32.6	6311.79	36.91	1.13229	484	16.814	50
657	19046.8	32.6	6348.70	37.13	1.13713	486	16.864	50
656	19079.4	32.6	6385.83	37.36	1.14199	489	16.914	50
655	19112.0	32.7	6423.19	37.59	1.14688	491	16.964	50
654	19144.7	32.7	6460.78	37.82	1.15179	493	17.014	50
653	19177.4	32.8	6498.60	38.05	1.15672	495	17.064	50
652	19210.2	32.8	6536.65	38.28	1.16167	497	17.114	50
651	19243.0	32.9	6574.93	38.49	1.16664	500	17.164	50

TABLE I—CONTINUED.

V	S_v	*Diff.*	A_v	*Diff.*	I_v	*Diff.*	T_v	*Diff.*
650	19275.9	32.9	6613.42	38.72	1.17164	502	17.214	51
649	19308.8	33.0	6652.14	38.95	1.17666	504	17.265	51
648	19341.8	33.0	6691.09	39.18	1.18170	507	17.316	51
647	19374.8	33.1	6730.27	39.41	1.18677	509	17.367	51
646	19407.9	33.1	6769.68	39.64	1.19186	511	17.418	51
645	19441.0	33.2	6809.32	39.88	1.19697	514	17.469	51
644	19474.2	33.2	6849.20	40.11	1.20211	517	17.520	52
643	19507.4	33.3	6889.31	40.34	1.20728	519	17.572	52
642	19540.7	33.3	6929.65	40.57	1.21247	522	17.624	52
641	19574.0	33.4	6970.22	40.80	1.21769	525	17.676	52
640	19607.4	33.4	7011.02	41.04	1.22294	527	17.728	53
639	19640.8	33.5	7052.06	41.29	1.22821	528	17.781	53
638	19674.3	33.6	7093.35	41.54	1.23349	530	17.834	53
637	19707.9	33.6	7134.89	41.79	1.23879	532	17.887	53
636	19741.5	33.7	7176.68	42.04	1.24411	534	17.940	53
635	19775.2	33.7	7218.72	42.29	1.24945	537	17.993	53
634	19808.9	33.7	7261.01	42.54	1.25482	540	18.046	53
633	19842.6	33.8	7303.55	42.79	1.26022	544	18.099	53
632	19876.4	33.9	7346.34	43.04	1.26566	547	18.152	53
631	19910.3	34.0	7389.38	43.22	1.27113	550	18.205	54
630	19944.3	34.0	7432.60	43.55	1.27663	552	18.259	54
629	19978.3	34.0	7476.15	43.81	1.28215	554	18.313	54
628	20012.3	34.1	7519.96	44.07	1.28769	557	18.367	54
627	20046.4	34.1	7564.03	44.33	1.29326	559	18.421	55
626	20080.5	34.1	7608.36	44.59	1.29885	562	18.476	55
625	20114.6	34.2	7652.95	44.84	1.30447	565	18.531	55
624	20148.8	34.3	7697.79	45.10	1.31012	567	18.586	55
623	20183.1	34.4	7742.89	45.36	1.31579	570	18.641	55
622	20217.5	34.5	7788.25	45.62	1.32149	573	18.696	55
621	20252.0	34.5	7833.87	45.88	1.32722	576	18.751	55
620	20286.5	34,5	7879.75	46.17	1.33298	579	18.806	56
619	20321.0	34.5	7925.92	46.46	1.33877	581	18.862	56
618	20355.5	34.6	7972.38	46.75	1.34458	584	18.918	56
617	20390.1	34.7	8019.13	47.04	1.35042	587	18.974	56
616	20424.8	34.8	8066.17	47.32	1.35629	590	19.030	57
615	20459.6	34.8	8113.49	47.60	1.36219	593	19.087	57
614	20494.4	34.9	8161.09	47.89	1.36812	596	19.144	57
613	20529.3	34.9	8208.98	48.17	1.37408	599	19.201	57
612	20564.2	34.9	8257.15	48.45	1.38007	602	19.258	57
611	20599.1	35.0	8305.60	48.73	1.38609	604	19.315	57

TABLE I—Continued.

V	S_v	*Diff.*	A_v	*Diff.*	I_v	*Diff.*	T_v	*Diff.*
610	20634.1	35.1	8354.33	49.00	1.39213	607	19.372	58
609	20669.2	35.1	8403.33	49.30	1.39820	610	19.430	58
608	20704.3	35.2	8452.63	49.61	1.40430	613	19.488	58
607	20739.5	35.3	8502.24	49.92	1.41043	617	19.546	58
606	20774.8	35.4	8552.16	50.23	1.41660	620	19.604	58
605	20810.2	35.4	8602.39	50.54	1.42280	623	19.662	58
604	20845.6	35.5	8653.93	50.85	1.42903	626	19.720	59
603	20881.1	35.5	8704.78	51.16	1.43529	629	19.779	59
602	20916.6	35.5	8755.94	51.46	1.44158	632	19.838	59
601	20952.1	35.5	8807.40	51.77	1.44790	635	19.897	59
600	20987.6	35.6	8858.17	52.22	1.45425	638	19.956	60
599	21023.2	35.7	8910.39	52.51	1.46063	642	20.016	60
598	21058.9	35.8	8962.90	52.80	1.46705	645	20.076	60
597	21094.7	35.9	9015.70	53.09	1.47350	648	20.136	60
596	21130.6	36.0	9068.79	53.38	1.47998	651	20.196	60
595	21166.6	36.0	9122.17	53.67	1.48649	655	20.256	60
594	21202.6	36.1	9175.84	53.96	1.49304	658	20.316	61
593	21238.7	36.1	9229.80	54.25	1.49962	661	20.377	61
592	21274.8	36.2	9284.05	54.55	1.50623	665	20.438	61
591	21311.0	36.2	9338.60	54.84	1.51288	668	20.499	61
590	21347.2	36.3	9393.44	55.28	1.51956	672	20.560	62
589	21383.5	36.3	9448.72	55.64	1.52628	674	20.622	62
588	21419.8	36.3	9504.36	55.99	1.53302	678	20.684	62
587	31456.1	36.4	9560.35	56.35	1.53980	682	20.746	62
586	21492.5	36.5	9616.70	56.71	1.54662	686	20.808	62
585	21529.0	36.6	9673.41	57.07	1.55348	689	20.870	63
584	21565.6	36.7	9730.48	57.43	1.56037	692	20.933	63
583	21602.3	36.8	9787.91	57.79	1.56729	696	20.996	63
582	21639.1	36.8	9845.70	58.15	1.57425	700	21.059	63
581	21675.9	36.8	9903.85	58.51	1.58125	703	21.122	63
580	21712.7	36.9	9962.36	58.87	1.58828	707	21.185	64
579	21749.6	37.0	10021.23	59.22	1.59535	710	21.249	64
578	21786.6	37.0	10080.45	59.58	1.60245	714	21.313	64
577	21823.6	37.1	10140.03	59.94	1.60959	718	21.377	65
576	21860.7	37.1	10199.97	60.31	1.61677	722	21.442	65
575	21897.8	37.2	10260.28	60.69	1.62399	725	21.507	65
574	21935.0	37.3	10320.97	61.06	1.63124	729	21.572	65
573	21972.3	37.3	10382.03	61.43	1.63853	733	21.637	65
572	22009.6	37.4	10443.46	61.79	1.64586	737	21.702	65
571	22047.0	37.5	10505.25	62.17	1.65323	741	21.767	65

TABLE I—Continued.

V	S_v	*Diff.*	A_v	*Diff.*	I_v	*Diff.*	T_v	*Diff.*
570	22084.5	37.6	10567.42	62.61	1.66064	745	21.832	66
569	22122.1	37.6	10630.03	63.03	1.66809	749	21.898	66
568	22159.7	37.7	10693.06	63.42	1.67558	753	21.964	66
567	22197.4	37.8	10756.48	63.83	1.68311	757	22.030	67
566	22235.2	37.8	10820.31	64.23	1.69068	760	22.097	67
565	22273.0	37.9	10884.54	64.62	1.69828	764	22.164	67
564	22310.9	37.9	10949.16	65.00	1.70592	769	22.231	67
563	22348.8	38.0	11014.16	65.37	1.71361	773	22.298	68
562	22386.8	38.1	11079.53	65.73	1.72134	777	22.366	68
561	22324.9	38.2	11145.26	66.11	1.72911	781	22.434	68
560	22463.1	38.3	11211.37	66.50	1.73692	785	22.502	68
559	22501.4	38.3	11277.87	66.97	1.74477	790	22.570	69
558	22539.7	38.3	11344.84	67.43	1.75267	794	22.639	69
557	22578.0	38.4	11412.27	67.89	1.76061	798	22.708	69
556	22616.4	33.5	11480.16	68.34	1.76859	802	22.777	69
555	22654.9	38.5	11548.50	68.79	1.77661	807	22.846	70
554	22693.4	38.6	11617.29	69.24	1.78468	811	22.916	70
553	22732.0	38.7	11686.53	69.69	1.79279	816	22.986	70
552	22770.7	38.8	11756.22	70.14	1.80095	820	23.056	70
551	22809.5	38.9	11826.36	70.59	1.80915	825	23.126	70
550	22848.4	39.0	11896.95	71.04	1.81740	829	23.196	71
549	22887.4	39.0	11967.99	71.50	1.82569	833	23.267	71
548	22926.4	39.1	12039.49	71.96	1.83402	838	23.338	71
547	22965.5	39.1	12111.45	72.41	1.84240	843	23.409	72
546	23004.6	39.2	12183.86	72.87	1.85083	847	23.481	72
545	23043.8	39.3	12256.73	73.33	1.85930	852	23.553	72
544	23083.1	39.3	12330.06	73.79	1.86782	857	23.625	72
543	23122.4	39.4	12403.85	74.25	1.87639	862	23.697	73
542	23161.8	39.5	12478.10	74.71	1.88501	866	23.770	73
541	23201.3	39.6	12552.81	75.27	1.89367	871	23.843	73
540	23240.9	39.6	12628.08	75.77	1.90238	876	13.916	74
539	23280.5	39.7	12703.85	76.26	1.91114	881	23.990	74
538	23320.2	39.8	12780.11	76.75	1.91995	886	24.064	74
537	23360.0	39.9	12856.86	77.23	1.92881	891	24.138	74
536	23399.9	39.9	12934.09	77.71	1.93772	896	24.212	75
535	23439.8	40.0	13011.80	78.19	1.94668	901	24.287	75
534	23479.8	40.1	13089.99	78.67	1.95569	906	24.362	75
533	23519.9	40.2	13168.66	79.15	1.96475	911	24.437	75
532	23560.1	40.3	13247.81	79.63	1.97386	916	24.512	76
531	23600.4	40.3	13327.44	80.11	1.98302	921	24.588	76

TABLE I—CONTINUED.

V	S_v	*Diff.*	A_v	*Diff.*	I_v	*Diff.*	T_v	*Diff.*
530	23640.7	40.4	13407.55	80.60	1.99223	927	24.664	76
529	23681.1	40.4	13488.15	81.12	2.00150	932	24.740	77
528	23721.5	40.5	13569.27	81.67	2.01082	937	24.817	77
527	23762.0	40.6	13650.94	82.24	2.02019	942	24.894	77
526	23802.6	40.7	13733.18	82.84	2.02961	947	24.971	77
525	23843.3	40.8	13816.02	83.45	2.03908	953	25.048	78
524	23884.1	40.8	13899.47	84.07	2.04861	959	25.126	78
523	23924.9	40.9	13983.54	84.70	2.05820	964	25.204	78
522	23965.8	41.0	14068.24	85.37	2.06784	970	25.282	78
521	24006.8	41.1	14153.61	86.05	2.07754	976	25.360	79
520	24047.9	41.2	14239.66	86.55	2.08730	981	25.439	79
519	24089.1	41.2	14326.21	87.07	2.09711	987	25.518	79
518	24130.3	41.3	14413.28	87.59	2.10698	993	25.597	80
517	24171.6	41.4	14500.87	88.11	2.11691	998	25.677	80
516	24213.0	41.5	14588.98	88.64	2.12689	1004	25.757	80
515	24254.5	41.6	14677.62	89.14	2.13693	1011	25.837	81
514	24296.1	41.7	14766.76	89.66	2.14704	1017	25.918	81
513	24337.8	41.7	14856.42	90.18	2.15721	1022	25.999	82
512	24379.5	41.8	14946.60	90.70	2.16743	1027	26.081	82
511	24421.3	41.9	15037.30	91.22	2.17770	1033	26.163	83
510	24463.2	42.0	15128.52	91.74	2.18803	1040	26.246	83
509	24505.2	42.1	15220.26	92.37	2.19843	1046	26.329	83
508	24547.3	42.2	15312.63	93.02	2.20889	1052	26.412	83
507	24589.5	42.2	15405.65	93.67	2.21941	1059	26.495	83
506	24631.7	42.3	15499.32	94.43	2.23000	1065	26.578	84
505	24674.0	42.4	15593.75	95.23	2.24065	1071	26.662	84
504	24716.4	42.4	15688.98	96.08	2.25136	1077	26.746	84
603	24758.8	42.5	15785.06	96.98	2.26213	1084	26.830	85
502	24801.3	42.6	15882.04	97.97	2.27297	1091	26.915	85
501	24843.9	42.7	15980.01	99.02	2.28388	1097	27.000	85
500	24886.6	42.8	16079.03	100.08	2.29485	1104	27.085	86

TABLE II.

Value of C for Service Guns, U. S. Navy.

Caliber, d.	*Gun.*	*W.*	*C.*	*C.*	$\frac{C}{2}$
Inches.	*Class.*	*Lbs.*	*Log.*	*Co-log.*	*Log.*
0.315	Small arms	0.03394	9.53409—10	0.46591	9.23306—10
0.45	Small arms	0.0686	9.52990—10	0.47010	9.22887—10
1.46	R. F	1.1	9.71268—10	0.28732	9.41165—10
1.85	R. F	3.3	9.98417—10	0.01583	9.68314—10
2.24	R. F	6.0	0.07765	9.92235—10	9.77662—10
3.00	Boat Gun	7.0	9.89086—10	0.10914	9.58983—10
4.00	R. F	33.0	0.31439	9.68561—10	0.01336
5.00	R. F	50.0	0.30103	9.69897—10	0.00000
5.00	B. L. R	60.0	0.38021	9.61979—10	0.07918
6.00	B. L. R	100.0	0.44370	9.55630—10	0.14267
8.00	B. L. R	250.0	0.59176	9.40824—10	0.29073
10.00	B. L. R	500.0	0.69897	9.30103—10	0.39794
12.00	B. L. R	850.0	0.77106	9.22894—10	0.47003
13.00	B. L. R	1100.0	0.81350	9.18650—10	0.51247
16.00	B. L. R	2000.0	0.89279	9.10721—10	0.59176